ELECTRIC CONDUCTIVITY
OF WEAKLY IONIZED GASES

By

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
April, 1967

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ACKNOWLEDGMENTS

The author wishes to express his appreciation to the Institute for Defense Analyses and the Advanced Research Projects Agency for the partial support which formed the foundation of this research. The author acknowledges gratefully the encouragement of Professor Alex E.S. Green and the conscientious typing by Mrs. Lynda Browne.

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Abstract of Dissertation Presented to the Graduate Council
in Partial Fulfillment of the Requirements for the
Degree of Doctor of Philosophy

ELECTRIC CONDUCTIVITY OF WEAKLY IONIZED GASES

In calculating the electric conductivity of gases and plasmas, it is not sufficient to define only the effective collision frequency. Therefore, two independent quantities are defined which permit simple adaptation of existing solutions of Maxwell equations (at constant collision frequency) to any realistic plasma in the presence of static magnetic fields. A detailed review of the theory of electric conductivity of weakly ionized gases herein is presented and supplemented with many additional derivations and computations. Included are tables of the effective quantities needed for the electric conductivity for an idealized single-component weakly ionized gas whose collision frequency has a power law dependence on the velocity (with the exponent in the range of -3 to +3). The average electron-neutral collision frequency was computed on the basis of latest reviewed cross sections for N_2, O_2, N, O, NO, air, He, Ar, Kr and Xenon.

[illegible]

[illegible] an el[illegible] Lorentz [illegible][2-4] [illegible] then many more [illegible]tions [illegible] Langevin,[5] Chapman and [illegible][10] Margenau,[22] etc. [illegible] yielding answers very closely [illegible] to Lorentz's simple ex[illegible]. Although these formulae [illegible] from each other only slightly, the differences have in many [illegible] a very pronounced effect on the propagation of electroma[illegible] radiation through plasma (e.g., in the propagation of radio waves in the ionosphere, in the microwave diagnostics of pl[illegible] laboratory, in the transmission of radar through the [illegible] in the transmission of electromagnetic signals [illegible] re-entering the earth's atmosphere at hyp[illegible] the interpretation of radar waves sca[illegible] to

We start [illegible] derivation, supplemented by the [illegible] necessary in the light of recent [illegible] follow with the section on the [illegible] weakly ionized plasma in which [illegible] solution [illegible]

[illegible] tromagneti[illegible] without [illegible]

The en[illegible]

[illegible] methods o[illegible]

[illegible]

where [illegible] of ele[illegible] ele[illegible] [illegible] namel[illegible]

$$\alpha_1 = \frac{[illegible]}{[illegible]}, \quad \frac{(\nu_p/\nu)_{eff}}{\omega_p/\nu} \quad [illegible]$$

[illegible] to plas[illegible]

[illegible]

More [illegible]

solutio[illegible]

[illegible] anti[illegible]

[illegible]

[illegible] having an electron [illegible] simple power law of [illegible] there have been only a few sporadic computations using single integrals, some integrals etc. This is generalized [illegible] on average collision frequency [illegible] realistic cross sections and also for a multicomponent plasma.

Based on review of experimental electron neutral elastic cross-sections for momentum transfer of various atoms and molecules the corrections a_1 and a_2 are computed, together with the average collision frequency, for weakly ionized He,Ar,Kr,Xe,N,O,NO and dry air from 100°K and 10000°K (with the application to the atmosphere, below 90 km and above 60 km, atomic oxygen is included in the calculations). After including contributions from electron elastic cross-sections with N, O and NO the tables are extended to weakly ionized air in an equilibrium up to 10,000°K and for few selected [illegible]. The average collision frequency is presented [illegible] for gases which are not in equilibrium.

Some of [illegible] this paper was first [illegible] the Spring of [illegible] seminar at the University of Florida on [illegible] were [illegible] supported [illegible] by the [illegible] the book, [illegible] [illegible]

1 MEAN [illegible] LORENTZ [illegible]

[illegible] lectures [illegible] [illegible] [illegible] considered [illegible] [illegible] or a [illegible] [illegible] rons under [illegible] [illegible] In the [illegible] he notion of [illegible] romagne [illegible] free [illegible]

$$m_e \frac{d\vec{v}_i}{dt} = [illegible]$$

[illegible] is the [illegible] [illegible] [illegible] [illegible] of the [illegible]

[illegible]

[illegible] frequency of the electromagnetic wave

[illegible] Eq. (1), neglecting [illegible]

$$[illegible] \frac{d\vec{v}_j}{dt} [illegible] + C_j + \frac{-e\vec{E}_0}{im_e\omega} (e^{-i} [illegible]$$

The [illegible] constant C_j is so [illegible] [illegible]ed a [illegible] est collision, [illegible] aged over all el[illegible] (j = 1 to N_e) is zero. [illegible] random (thermal) [illegible] superimposed [illegible] component of [illegible] the direction of [illegible] field $\vec{E}$ [illegible] c [illegible] heavy ions) ar[illegible] thermal [illegible] $v_{thermal}$ [illegible] that at t [illegible], all dir[illegible] ment and [illegible] are equally probable [illegible] then in order [illegible] obtain mean values [illegible] [illegible]ns, we find

$$\bar{v}_e \; \frac{1}{N_e} \sum_{j=1} v_j \; \frac{ie\vec{E}_0}{m_e\omega} e^{-i\omega t} \sum_{j=1}^{N_e} \Big[1 \; [illegible]$$

where

$$\theta_j \; t - t \; [illegible]$$

This is the mean value of the [illegible] definite instant, [illegible] low at time [illegible] still be moving wi[illegible] [illegible] city independent collision frequency of [illegible] particles and [illegible] 1/ [illegible] is the average [illegible]

[illegible] is [illegible]

the time interval $\Delta\theta(\Delta\theta$ [illegible]), and [illegible] number of [illegible] that will (has probability of) collide [illegible] out of the [illegible] group of N_o electrons. Therefore, [illegible] is changed by

$$dN_o = - N_o \bar{\nu} d\theta \quad ,$$

which integrates to the number of electrons remaining in [illegible] at the time θ

$$[illegible]\, e^{-\bar{\nu}\theta}$$

The number of electrons that will collide between time θ and [illegible] is thus given by

$$dN = - [illegible]\, e^{-\bar{\nu}\theta} d\theta$$

The fraction from the freely traveling electrons [illegible] originally selected [illegible] which will collide in [illegible] θ [illegible] $d\theta$ is

$$[illegible]\, e^{-\bar{\nu}\theta} d\theta$$

[illegible] will [illegible] and also the [illegible] [illegible] the time θ and [illegible]

[illegible] originally selected [illegible]

$$[illegible]\, e^{-\bar{\nu}}\, d\theta$$

[illegible] pr[illegible]

[illegible] will [illegible]

---

electrons freely traveling from 0 to [illegible]

$$\bar{\bar{v}} = \int_0^\infty (\vec{v})_{N_1} \nu e^{-\bar{\nu}\theta} \text{[illegible]}$$

We could do the calculations at a differen[illegible] different number of N_2 electrons colliding [illegible] follow in time those out of this group whic[illegible] since our solutions to Eqs (6) and (8) are independent [illegible] ber N_1 and the time t_1, we can conclude that these values [illegible] deed the proper average quantities. The mean value of [illegible] tory (drift) velocity of an electron is

$$\overline{(\vec{v})} = \int_0^\infty (\vec{v})_{N_1} \bar{\nu} e^{-\bar{\nu}\theta} d\theta = \frac{e\vec{E}_0 e^{i\omega t}}{m_e(\bar{\nu} - i\omega)}$$

and in the real form

$$\overline{|\vec{v}|} \quad Re(\overline{\vec{v}}) \quad \frac{e\vec{E}_0}{m_e\sqrt{\omega^2+\bar{\nu}^2}} \cos(\omega t - \beta) = \frac{e\vec{E}_0}{m_e} \frac{\bar{\nu}\cos \text{[illegible]}}{\text{[illegible]}}$$

where

$$\beta \quad \text{arc tan} \frac{\omega}{\bar{\nu}}$$

This mean velocity of a free electron was derived pre[illegible] Appleton and Chapman,[illegible] who followed closely the deriva[illegible] and has bee[illegible] discussed [illegible] recently by others [illegible] independent collision freq[illegible]ncy and a weak field [illegible] one can obtain the same res[illegible]lt from [illegible] neglecting terms of order [illegible] etc.[5-13]

One can now ask which differential eq[illegible] same average res[illegible] what form has the [illegible] which will describe an average o[illegible] that the average velocity $\overline{(v)}$ sa[illegible] normal equation

$$m_e \frac{d\overline{(v)}}{dt} + m_e \nu \overline{(v)}$$ [illegible]

Dealing only with average [illegible] we can interpret ν as a coefficient of dynamic friction and the term $-\nu m_e \overline{(v)}$ in Eq. (5) as the frictional type damping force on an electron ca[illegible] by electron collisions, proportional to [illegible] of an electron [illegible] the direction opposite to it, [illegible] $\frac{F}{m}$ [illegible] interpreted as the average force on a free electron resulting from an applied time harmonic electric field.

In the next sections we will discuss [illegible] averages, plasma frequency and electric conductivity at constant collision frequency

[illegible]

[illegible] we will [illegible] average quantities over time τ, [illegible] We have [illegible] $\frac{1}{\tau}\int_0^{\tau} (\sin^2\omega t)\, dt \simeq 1/2$, [illegible] cos ωt) [illegible] Thus the root mean square velocity is from Eq (15),

$$[illegible] = \sqrt{\langle [\mathrm{Re}\,(\overline{v})]^2 \rangle} = \frac{1}{\sqrt{2}} \; \frac{eE_0}{m_e\sqrt{\omega^2 + \overline{\nu}^2}} = \frac{1}{\sqrt{2}} \; |\overline{v}|_{max} \qquad (16)$$

The average kinetic energy imparted to the electron from the field (drift energy) is

$$[illegible] \langle \mathrm{Re}\,(eE) \quad \mathrm{Re}\,(\overline{v})\rangle/\overline{\nu} = \langle eE_0 \cos \omega t)|\overline{v}|\rangle \;, \qquad (17)$$

[illegible] after substitution of $\overline{v}$ from Eq (8), gives

$$[illegible] \frac{1}{4} \; \frac{e^2E_0^{\,2}}{m_e(\omega^2 + \overline{\nu}^2)} = m_e\, u^2 = \frac{1}{2}\, m_e \; |\overline{v}|^2_{max} \qquad (18)$$

[illegible] c shown also from the Boltzmann equation (see Appendix [illegible] ω0)

PLA[illegible]

[illegible] homogeneous plasma [illegible] electrons per cm³ along some configuration [illegible] restoring electric field $\vec{E}$ will be set up, which can be obtained from one of Maxwell's equations

$$\nabla \cdot \vec{E} = 4\pi e N_e \tag{19}$$

Just displacing electrons the same distance along one dimension in an infinite homogeneous slab (the ions are assumed to be stationary)

$$\frac{dE_x}{dx} = 4\pi e N_e \;, \quad E_x = 4\pi e N_e x \;, \tag{20}$$

where E_x is the electric field arising from the displacement of electrons. Displacing the same distance radially N_e electrons in an infinite, homogeneous plasma along an infinite cylinder in a radial direction, and using cylindrical coordinates

$$\nabla \cdot \vec{E} = \frac{1}{r}\frac{\partial}{\partial r}(rE_r) = \frac{\partial E_r}{\partial r} + \frac{1}{r}E_r = 4\pi e N_e \;,$$

we get

$$E_r = \frac{4\pi e N_e}{2} r \;. \tag{21}$$

Displacing N_e electrons in an infinite homogeneous plasma along the radius of a sphere and using spherical coordinates we get

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} (r^2 E_r) - \frac{\partial E_r}{\partial r} + \frac{2}{r} E_r - 4\pi N_e \, ,$$

with a solution

$$E_r = \frac{4\pi e N_e}{3} r \qquad (22)$$

In all the above three cases, E was of the type

$$E = \frac{4\pi e N_e}{a} r \qquad (23)$$

The equation of motion of an electron (neglecting collisions) after the displacement to the distance r_{max} at time t = 0 is [15,16] (plasma electron oscillation)

$$m_e \frac{d^2 r}{dt^2} = - eE = - e \frac{4\pi e N_e}{a} r \, ,$$

or

$$\frac{d^2 r}{dt^2} + \frac{\omega_p^2}{a} r = 0 \, , \quad \omega_p^2 = \frac{4\pi e^2 N_e}{m_e} \qquad (24)$$

with a solution

$$r = r_{max} \cos \frac{\omega_p}{\sqrt{a}} t$$

$$v = - r_{max} \frac{\omega_p}{\sqrt{a}} \quad t \qquad (25)$$

The (angular) plasma frequency[15] of an electron is given

$$\boxed{\omega_p = \sqrt{4\pi N_e e^2/m} = 5.64 \times 10^4 \sqrt{N_e} \qquad \text{sec}^{-1}}$$

$\omega_p = 2\pi f_p$,

$f_p = 8.978 \times 10^3 \sqrt{N_e}$ cycles/sec ,

where N_e stands for the number of electrons in 1 cm³, m_e is the mass of the electron, e is the charge on the electron. Thus the electrons, neglecting collisions, will continue to oscillate with an angular frequency $\omega_p/\sqrt{\alpha}$ along an equilibrium point, the slab with angular frequency ω_p, the cylinder with $\omega_p/\sqrt{2}$, and the sphere with $\omega_p/\sqrt{3}$.[16] Taking collisions into account, the resulting oscillations will be damped, transferring some of the electromagnetic energy into energy of plasma (temperature increase, atomic excitation, etc.)

Often in the literature plasma frequency is referred to as Langmuir frequency, since Tonks and Langmuir were first to introduce it

[illegible] MAGNETIC CO[illegible] PLASMA
[illegible] ABSENCE OF MAGNE[illegible]

The [illegible] formula for the (complex) [illegible] absence of an external magnetic field [illegible] and from the definition of σ [illegible]

$$\vec{j} = \sigma\vec{E} \quad , \quad Re(\vec{j}) = \frac{N_e e^2/m_e \vec{E}_0}{\bar{\nu}^2 + \omega^2} (\bar{\nu} \cos [illegible]$$

where $\vec{j}$ is the current density and $Re(\vec{j})$ is the real part of [illegible]

$$\vec{j} = \sigma\vec{E} = N_e e(\overline{\vec{v}})$$

and

$$\sigma = \frac{N_e e^2/m_e}{\bar{\nu} - i\omega} \quad ,$$

or

$$4\pi\sigma' = \frac{\omega_p^2}{\bar{\nu} - i\omega} = \sigma_r + i\sigma_i$$

where we introduced σ′, because 4πσ will appear in Maxwell eq [illegible]

$$\sigma_r = \frac{\omega_p^2 \bar{\nu}}{\omega^2 + \bar{\nu}^2} \quad ,$$

$$\sigma_i = \frac{\omega_p^2 \omega}{\omega^2 + \bar{\nu}^2} \quad ,$$

and where ω is the incident angular frequency of electromagnetic wave, ω = 2πf, $\bar{\nu}$ is the so-called average collision frequency, i e , average number of collisions which an electron makes with other particles, σ_r and σ_i are the real and imaginary parts of complex conductivity, respectively (We are neglecting here collision of ions with ions, electrons with electrons, motion of ions and dependence of ν on electron velocity)

The specific impedance is usually defined as

$$Z(\omega) = \frac{1}{\sigma} = R - iX ,$$

where

$$R = \frac{4\pi\nu}{\omega_p^2}$$

is called the resistivity and

$$X = \frac{4\pi\omega}{\omega_p^2}$$

is called the reactance

Using the electric charge in electrostatic units, m_e in grams, N_e in cm^{-3}, we obtain σ in sec^{-1}, which we call σ_{esu}, or $\sigma(\mathrm{sec}^{-1})$. Conductivity in other units are related as follows,

$$\sigma_{esu} = \sigma\left(\frac{\mathrm{cm}^{-3}(\mathrm{esu})^2}{\mathrm{g\ sec}^{-1}}\right) = \sigma(\mathrm{sec}^{-1}) = 9\mathrm{x}10^9\ \sigma(\mathrm{mho/m}) = 9\mathrm{x}10^{20}\ \sigma(\mathrm{abmho/cm})$$

$$\sigma_{MKS} = \sigma\left(\frac{\mathrm{m}^{-3}\ \mathrm{Coul}^2}{\mathrm{kg\ sec}^{-1}}\right) = \sigma(\mathrm{ohm}^{-1}\ \mathrm{m}^{-1}) = \sigma(\mathrm{mho/m}) = 10^9\sigma(\mathrm{abmho/m})$$

$$= \frac{1}{9 \times 10^9}\sigma_{esu} = 1.11 \times 10^{-10}\ \sigma_{esu}$$

$$\sigma(\mathrm{mho/cm}) = 10^{-2}\ \sigma(\mathrm{mho/m}) = \left(\frac{1}{9 \times 10^{11}}\right)\sigma_{esu} = 10^9\sigma(\mathrm{abmho/cm})$$

The complex notation introduced previously for the electric field, electric conductivity and current density is a notation of a convenience and one should bear always in mind that the electric field and current density and other directly measurable physical quantities are always real. Nevertheless, in taking products of complex quantities one must be very careful, always when in doubt returning to the operations with real quantities only. Let us as an illustration obtain $\vec{j}$ and $\vec{j}\cdot\vec{E}$, where j is the electron current density.

Working with real quantities

$$m_e\dot{\vec{v}} + m_e\nu\vec{v} = e\vec{E}_o \cos \omega t$$

$$\vec{v} = \frac{e\vec{E}_o}{m_e\sqrt{\omega^2+\nu^2}} \cos(\omega t - \varphi)$$

$$= \frac{e\vec{E}_o}{m_e(\omega^2+\nu^2)}(\nu \cos \omega t + \omega \sin \omega t)$$

$$\varphi = \mathrm{arc\ tan}\ \omega/\nu$$

Note that the complex $\vec{v}$ is out of phase by φ from $\vec{E}$

$$\vec{j} = N_e e\vec{v} = \sigma\vec{E}$$

$$\vec{j}\cdot\vec{E} = N_e \frac{e^2E_o^2[\nu\cos^2\omega t + \omega \sin \omega t \cos \omega t]}{m_e(\omega^2+\nu^2)}$$

with complex quantities (see Eq. (13))

$$m_e\dot{\vec{v}} + m_e\nu\vec{v} = e\vec{E}_o e^{-i\omega t}$$

$$\vec{v} = \frac{e\vec{E}_o e^{-i\omega t}}{m_e(\nu - i\omega)}$$

$$\mathrm{Re}(\vec{v}) = \frac{e\vec{E}_o(\nu\cos\omega t + \omega\sin\omega t)}{m_e(\omega^2+\nu^2)}$$

$$\vec{j} = \mathrm{Re}(\sigma\vec{E})$$

$$\mathrm{Re}(\vec{j}\cdot\vec{E}) = N_e \frac{e^2E_o^2(\nu\cos 2\omega t + \omega s \text{[illegible]})}{m_e(\omega^2+\nu^2)}$$

Which is wrong. The correct expression is

$$\vec{j}\cdot\vec{E} = (\mathrm{Re}\ \vec{j})\cdot(\mathrm{Re}\ \vec{E})$$

The *electric resistivity* is [illegible] real conductivity, $\eta = 1/\sigma_r$ for the direct current (DC) conductivity (conductivity for $\omega = 0$). The Lorentz formula gives, in this case,

$$\sigma_{DC} = \frac{\omega_p^2}{\nu} \quad , \qquad (\sigma_i)_{DC} = 0 \qquad (32)$$

The (complex) electron *mobility* is defined by the relation

$$\langle\vec{v}\rangle = \mu_e \vec{E} \quad ,$$

where $\langle\vec{v}\rangle$ is the electron drift velocity. From Eq. (8),

$$\mu_e = \frac{e/m_e}{\nu - i\omega} \quad \text{or} \quad \sigma = N_e e \mu_e \qquad (33)$$

3. THE ELECTRIC CONDUCTIVITY OF WEAKLY IONIZED GA[illegible] IN THE ABSENCE OF EXTERNAL MAGNETIC FIELDS (WEAK ELECTRIC FIELDS) (MAJUMDAR - MARGENAU THEORY)

Majumdar[18] derived the transport coefficie[illegible] a electric conductivity (or actually mobility) [illegible] the Boltzmann transport equation when colli[illegible] a function of electron velocity. In the absence of [illegible] magnetic field for a plasma with a given approximat[illegible] distribution $f_0^{(e)}$ of electrons (the same result wa[illegible] later independently by Margenau[19] for the electri[illegible] see Appendix II for derivation and also Allis[20])

$$\boxed{\sigma = -\frac{4\pi e^2}{3m_e}\int_0^\infty \frac{v^3}{\nu \quad 3\,p}\,\frac{\partial f_0^{(e)}}{\partial v}\,dv} \;, \quad \int_0^\infty f_0^{(c)}\,d\underline{v}$$

$$d\underline{v} \quad 4\pi v^2 dv\,,\ \nu \quad \nu(v)$$

[illegible] ν is the velocity dependent electron [illegible] with neutral atoms and molecule[illegible]

[illegible] and Enoch[1] carefully rederived Eq. ( ) [illegible] valid for spatially homoge[illegible] with [illegible]tropic elastic scattering and [illegible] the first order in [illegible]

[illegible] Walsh[illegible] and Marcus[illegible] [illegible]). We will [illegible]
(34) [illegible] old [illegible] [illegible]
por[illegible] [illegible]
w[illegible]ly [illegible]
and [illegible]

F[illegible] Ma[illegible] [illegible] T, where
[illegible] is the [illegible]

$$(a) \quad [illegible]$$

[illegible]

$$\boxed{[illegible]}$$

[illegible] stributi[illegible] [illegible] (ε'a)}, namely

$$[illegible]$$

[illegible] Eqs. (25) and (35a) [illegible] Eq. (3[illegible])
[illegible]

[illegible]

$$= \frac{m_e v^2}{kT_e}$$

[illegible]

$$[illegible] \int \frac{W\sqrt{W}\, e^{W}\, W}{\nu(W) - i\omega}\ \frac{\omega_p^2}{\omega}\left(I + i\ [illegible]\right)$$

[illegible] I_1 and I_2 for future [illegible] plasma, [illegible]

$$[illegible] \left(I_1 + i\ [illegible]\ I\right)$$

The real pa[illegible]tivity b[illegible]

$$[illegible] \int [illegible]\ W\sqrt{W}\, e^{-W}\, [illegible]$$

$$[illegible]\ \frac{4}{3\sqrt{\pi}}\, \omega_p \int \frac{1}{[illegible]}\ \nu\sqrt{W}\ ^{W}\, dW = \left(\frac{\omega_p}{\omega}\right)^2 \omega I_R$$

[illegible] the [illegible] t (see Ref 60), [illegible]

$$[illegible] = \frac{4}{3\sqrt{\pi}}\ \frac{\omega}{\nu} \int \frac{e^{-W} W\sqrt{W}\, dW}{1 + [illegible]}$$

The [illegible]

[illegible]

$$\text{[illegible]} = \frac{4}{3\sqrt{\pi}}\,\omega_p^2 \int_0^\infty \frac{1}{1\ \text{[illegible]}}\ \text{[illegible]}\ e^{-W}\,dW \qquad \text{[illegible]}$$

[illegible]

$$\text{[illegible]} = \frac{4}{3\sqrt{\pi}}\,\omega_p^2 \int_0^\infty \frac{(\nu/\omega)}{\omega^2 + \nu^2}\ \text{[illegible]}\ \sqrt{W}\,dW\ ,$$

[illegible]

respectively.

We see that [illegible] depend only [illegible] or the [illegible] ω_1/ν and ω/ν.

If ν is independent [illegible] $\left(\frac{\partial\nu}{\partial v}\ \text{[illegible]}\right)$ [illegible] from Eq.(25) and [illegible]

$$\int \text{[illegible]}\ \sqrt{W}\ \text{[illegible]}$$

[illegible]

[illegible]

[illegible]

[illegible]

$$\left(\frac{\omega_p}{\nu}\right) [illegible]$$

[illegible]

[illegible]

$$[illegible] \frac{1}{[illegible]\left[1 + \left(\frac{a}{\nu}\right)^2\right]}$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

for [illegible]

$$\frac{[illegible]}{[illegible]} \quad \frac{[illegible]}{[illegible]}\int^{\infty} e^{-W}\, W^{3/2}\, dW \ldots$$

[illegible]

$$\left(\frac{[illegible]}{[illegible]}\right)^{[illegible]} \sim 1 ,$$

[illegible] average collision frequency [illegible] this definition [illegible] throughout the rest of the book, i.e.,

$$\boxed{\begin{aligned}\bar{\nu} &= \frac{8}{3\sqrt{\pi}}\left(\frac{m_e}{2KT_e}\right)^{5/2}\int_0^{\infty} \nu(v)\, v^4 \exp(-[illegible])\, dv \\ &= \frac{4}{3\sqrt{\pi}}\int_0^{\infty} \nu(W)\, W^{3/2}\, e^{-W}\, dW \quad , \quad (\omega \gg \nu)\end{aligned}}$$

In Eqs. ([illegible]) and (53) the defined effective quantities [illegible] in terms of $\bar{\nu}$)

$$\left(\frac{\omega}{\nu}\right)_{eff} = \frac{\omega}{\bar{\nu}} \quad \left(\frac{\omega_p}{\nu}\right)_{eff} = \frac{[illegible]}{\bar{\nu}} \quad \left(\frac{[illegible]}{[illegible]}\right)_{[illegible]} \quad [illegible]$$

[illegible] → 1, α_2 → 1, α_3 → 1, as $\omega/\bar{\nu} \to \infty$.

[illegible] ([illegible] Eq. ([illegible])), [illegible]

$$[illegible]\, 4\pi\left(\frac{m_e}{2\pi kT_e}\right)^{[illegible]}\int_0^{\infty} \nu(v)\, v\, [illegible] \exp([illegible]) \ldots$$

[illegible] only for a [illegible] velocity [illegible] constant factor [illegible] $\bar{\nu}$ [illegible] Eqs. (181)-(184), e.g., [illegible] Phys. Fluids [illegible] (1964); 8, 1012 (1965)

For t[illegible] case wher [illegible]

$$\left.\begin{aligned} I_1 & [illegible] \frac{}{\sqrt{\pi}} \bar{\nu} \int_0 \frac{1}{\nu} W^{[illegible]} e^{-W}\, dW \\ I & [illegible] \frac{4}{3\sqrt{\pi}} \bar{\nu}^2 \int_0^{\infty} \frac{1}{\nu^2} W^{[illegible]} e^{-W}\, dW \end{aligned}\right\},$$

[illegible] (d[illegible] rrent case) from Eq (35) [illegible]

$$\left.\begin{aligned} & \boxed{\frac{}{I} \; \frac{1}{D} \; [illegible] \; \frac{1}{\nu_{DL}} \; \frac{4}{3\sqrt{\pi}} \int_0^{\infty} \frac{1}{\nu} W^{[illegible]} e^{-W}\, dW} \\ & \boxed{\frac{1}{\mu} \; \frac{}{\sqrt{\pi}} \left(\frac{m_e}{2kT_e}\right)^{[illegible]} \int_0^{\infty} \frac{v^4 \exp\left(-\frac{1}{2} m_e v^2/kT_e\right)}{\nu(v)}\, dv} \end{aligned}\right\} \quad ([illegible])$$

[illegible] hat f r a highly i n ized plasma Eqs (50) [illegible] e to the neglect of electron-election [illegible]

[illegible] define an average effective collisio[illegible] [illegible] an average effective collision fr[illegible] [illegible] the range of frequen ies (e g [illegible]) [illegible] u nearing to the average [illegible] [illegible] l only as a theoreti ally comput[illegible] [illegible] parameter in the equation) In that [illegible] meaningful parameters [illegible] and [illegible] (for

$_r$ and σ [illegible]

[illegible] and σ_2 [illegible]

paragraph. [illegible]

homogeneous [illegible]

$\gg v$ and [illegible]

[illegible] $_{DC}$ [illegible]

[illegible] the [illegible]

[illegible] or [illegible]

[illegible] to Eq. (34) [illegible]

[illegible] for $\mu \gg$ [illegible]

$$v = \frac{4\pi}{3N_e} \int [illegible]$$

EFFECTIVE ω/ν, ω_p/ν and ω_p/ω FOR ANY PLASMA IN THE ABSENCE OF EXTERNAL MAGNETIC FIELD

Let us define $(\omega/\nu)_{eff}$, $(\omega_p/\nu)_{eff}$ and $(\omega_p/\omega)_{eff}$ [illegible] [illegible] relations, in analogy to Eq. (43) and (44):

$$\epsilon_r = 1 - \left(\frac{\omega_p}{\nu}\right)^2 I_2 = 1 - \frac{(\omega_p/\nu)^2_{eff}}{\left(\frac{\omega}{\nu}\right)^2_{eff} + 1} = 1 - \frac{(\omega_p/\omega)^2_{eff}}{1 + \left(\frac{\omega}{\nu}\right)^2_{eff}}$$

and

$$\epsilon_i = \left(\frac{\omega_p^2}{\omega\nu}\right) I_1 = \frac{(\omega_p/\nu)^2_{eff}}{\left(\frac{\omega}{\nu}\right)_{eff}\left[1 + \left(\frac{\omega}{\nu}\right)^2_{eff}\right]} = \frac{(\omega_p/\omega)^2_{eff}}{\left(\frac{\omega}{\nu}\right)_{eff}\left[1 + \left(\frac{\omega}{\nu}\right)^2_{eff}\right]}$$

where I_1 and I_2 are integrals defined for any electron distribution functions. (I_1 and I_2 are quite general, not restricted to the previous theory for weakly ionized plasma.) Then we obtain

$$\left(\frac{\omega}{\nu}\right)_{eff} = \frac{\epsilon_i}{\epsilon_r} = \frac{\omega}{\nu}\frac{I_2}{I_1} = \frac{1-\epsilon_r}{\epsilon_i}$$

$$\left(\frac{\omega_p}{\nu}\right)_{eff} = \frac{\omega_p}{\nu} n_2$$

$$\left(\frac{\omega_p}{\omega}\right)_{eff} = \frac{\omega_p}{\omega} n_2 \frac{I_1}{I_2} ,$$

where

$$n_2 = \sqrt{I_2\left[1 + \left(\frac{\omega}{\nu}\frac{I_2}{I_1}\right)^2\right]}$$

[illegible] shown from Eq. (52) and Eq. (53) that (one should not [illegible] ϵ_r, $\epsilon_{r,eff}$ and $\epsilon_{i,eff}$ and the parentheses around effective quantities should not be discarded)

$$\left(\frac{\omega_p}{\nu}\right)_{\text{eff}} = \left(\frac{\omega_p}{\omega}\right)_{\text{eff}} \cdot \left(\frac{\omega}{\nu}\right)_{\text{eff}}$$

[illegible] the following quantities, [illegible]

$$\left.\begin{aligned} \alpha_1 &= \frac{(\omega/\nu)_{\text{eff}}}{\omega/\bar{\nu}} = \frac{I_2}{I_1} \\ \alpha_2 &= \frac{(\omega_p/\nu)_{\text{eff}}}{\omega_p/\nu} = \sqrt{I_2 \cdot \left[1 + \alpha_1^2\left(\frac{\omega}{\bar{\nu}}\right)^2\right]} \\ \alpha_3 &= \frac{(\omega_p/\omega)_{\text{eff}}}{\omega_p/\omega} = \alpha_2/\alpha_1 \ . \end{aligned}\right\}$$

and

[illegible] constant becomes (see Eqs.(42), (52) and [illegible]

$$\left.\begin{aligned} & \epsilon' = 1 - \frac{[illegible]}{[illegible] + \bar{\nu}} , \quad \epsilon' = \epsilon_r + i\epsilon_i \\ & \boxed{\epsilon_r = 1 - \frac{[illegible]}{[illegible] + \bar{\nu}^2}} \quad \boxed{\epsilon_i = \frac{\alpha_2\alpha_3\omega_p^2\,\bar{\nu}/\omega}{\alpha_1^2\omega^2 + \bar{\nu}^2}} \end{aligned}\right\} \quad (59)$$

[illegible] electr[illegible] [illegible]

$$\left.\begin{aligned} & \boxed{[illegible] = \sigma_r + \sigma_i} \\ & [illegible] \qquad \sigma_i = \frac{\alpha^2\omega_p^2\,\omega}{\nu^2 + \mu^2\alpha_1^2} \end{aligned}\right\} \quad (60)$$

The [illegible] α₁, α₂ and α₃ approach unity for an infinite [illegible] of [illegible] at [illegible] are from Lorentz' [illegible] will [illegible] on, we already defined [illegible]

$$= \frac{8}{3\sqrt{\pi}} \int_0^\infty \nu(W) W^{3/2} e^{-W}\, dW$$

which will be shown later is a good expression not only for a weakly ionized gas but also for a partially and fully ionized plasma (as long as $\omega >> \nu$) and a convenient normalization parameter for other frequencies (Ginsburg[6] calls this $\bar{\nu}$, ν_{eff}, his Eq. (68)).

At one time there was an effort[23,6] to have only one quantity ν_{eff}, but obviously it cannot be uniquely defined to insure compatibility, except only for $\omega >> \nu$ and for $\omega << \nu$.

We will later show how α_1, α_2, α_3 can also be used in the electric conductivity tensor, when the plasma is in a static magnetic field (Eq. (36), etc.).

The value of the here introduced, and later tabulated (Table A-1, Figs. C-1-C-9) effective corrections ($\alpha_1, \alpha_2, \alpha_3$) is that these allow us using any previously solved cases of propagation of electromagnetic waves through a given plasma by directly interpreting the used ω_p/ω and ω/ν as actually effective quantities. Thus, if somebody previously solved numerically homogeneous plasma slab using Lorentz' conductivity for $\omega_p/\omega = 1.0$ and $\omega/\nu = 0.09$ (with constant ν), we can still use the results for velocity dependent ν by interpreting the parameters as $(\omega_p/\omega)_{eff} = 1.0$ and $(\omega/\nu)_{eff} = 0.09$. Then, for example, for a particular plasma (e.g., $\nu \sim v^2$, Table A-1) for a given $\omega_p/\omega = 1/0.6741$ one obtains $\alpha_3 = 0.6741$ and $\omega/\bar{\nu} = 0.025$. Thus, the reflection, transmission, etc. of the plasma having $\nu \sim v^2$ and $\omega_p/\omega = 1/0.6741$ and $\omega/\bar{\nu} = 0.025$ are exactly the same as for a plasma having ν = constant, $\omega_p/\omega = 1$ and $\omega/\bar{\nu} = 0.09$.

As we have stated at the end of the previous section from experimental ε's, in a homogeneous plasma for all ω (except for ω >> ν and ω = 0) one can with the help of theoretical a's deduce ν̄ for ω >> ν̄. There is no significance to the average collision frequency at intermediate ω's. One can compare the measured ratio (as interpreted from microwave measurement for an assumed geometry of a homogeneous plasma)

$$\frac{\sigma_i^{exp}}{\sigma_r^{exp}} = \frac{1 - \epsilon_r^{exp}}{\epsilon_i^{exp}}$$

with the theoretical one,

$$\frac{1 - \epsilon_r}{\epsilon_i} = a_1 \frac{\omega}{\nu} , \qquad (62)$$

$$\bar{\nu} = a_1 \bar{\nu}^{(L)} ,$$

where $\bar{\nu}^{(L)}$ is the collision frequency obtained assuming Lorentz theory, or from the theoretical a_1 one can deduce semi-empirical ν̄ at ω >> ν. Note that the above ratio is independent of electron concentration. The other comparison, of course, is that of ϵ_r and ϵ_i directly with the computed ϵ_r and ϵ_i (e.g., through the use of tables of a's). From experimental ϵ_r and ϵ_i in a homogeneous plasma one can deduce electron concentration from theoretical a_3, i.e., from Eq. (59) taking the ratio of $1 - \epsilon_r$ to ϵ_i and substituting for $\bar{\nu}/\omega$ in $1 - \epsilon_r$ one obtains

$$\omega_p^2 \frac{1}{\omega^2} = \frac{\epsilon_i^2 + (1 - \epsilon_r)^2}{1 - \epsilon_r} , \qquad (63)$$

or

$$= \frac{1}{a_3} N_e^{(L)} ,$$

where $N_e^{(L)}$ is the electron concentration obtained assuming Lorentz' [illegible] Eqs.(62) and (63)were used by F. Tevelow[46] in comparing his experimental results obtained in a shock tube for high temperature air with our calculations (see Figs. C-7 to C-9)

[illegible]

[illegible]

$$\sigma' = \frac{\omega_p^2 \nu}{\omega^2 + \nu^2} I_1 + i \frac{\omega_p^2 \omega}{\omega^2 + \nu^2} I_2$$

[illegible]

$$\sigma = \frac{\omega_p^2}{\nu} I_1 - \frac{\omega_p^2 \nu}{\omega^2 + \nu^2} I_1$$

and

$$\sigma = \left(\frac{\omega_p}{\nu}\right)^2 I_2 = \frac{\omega_p^2 \nu}{\omega^2 + \nu^2}$$

where [illegible] constant collision frequency. [illegible] a weakly [illegible] plasma and Maxwellian distribution [illegible]

$$I_1 = \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{\omega^2 + \nu^2}{\omega^2 + \nu^2} x\, e^{-W} W^{3/2} dW$$

[illegible] given [illegible] (8) and [illegible]

$$I_2 = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{\omega^2 + \nu^2}{\omega^2 + \nu^2} e^{-W} W^{3/2} dW$$

For ν [illegible] furthermore [illegible] I_1 and I_2 are

$$I_1 = \frac{1}{(\,\cdot\,) + 1} h_0(x, \nu)$$

and

$$I_2 = \frac{\cdot}{(\,\cdot\,) + 1} g(x, \nu)$$

which are [illegible] for [illegible] 's [illegible] or a [illegible]

Shkarofsky[illegible] [illegible] h_σ, which appea[illegible]

$$\frac{\omega_b^{\;2}}{j\,\nu - i\omega h_\sigma}$$ [illegible]

[illegible] ion [illegible] rection facto[illegible] [illegible] llow[illegible] [illegible]lations[illegible]

$$I_1 \quad \frac{J_n}{J_{\pi}^{\;2} + \omega^2 h_\sigma^{\;2}}$$ [illegible]

[illegible]

[illegible]

$$\sqrt{\frac{\lambda_e}{1\ r}\,(1 + \omega_1\ [illegible]} \quad \frac{1}{\sqrt{h_\sigma}}$$ [illegible]

[illegible]

[illegible]

[illegible]

where [illegible]

[illegible]

and ν_{sc} are listed by us extensively for weakly ionized plasmas in Appendices D-1 and E and for fully ionized plasmas in Refs. 60, 61 and 62.

Molmud[35] defined a complex quantity $g = g_r + ig_i$ (in sec^{-1}) so that one could formally use Lorentz (Langevin) equation (see Eq. (1)),

$$m_e \frac{d\vec{v}}{dt} + m_e g\vec{v} = e\,\vec{E}_0\, e^{-i\omega t}$$

with

$$g' = \frac{\alpha_2\alpha_3\omega_p^2}{\nu - i\omega\alpha_1} - \frac{\omega_p^2}{g_r - i(\omega - g_i)} \quad . \tag{77}$$

Using our α's we can write

$$g = \frac{\bar{\nu}}{\alpha_2\alpha_3} + i\omega[1 - \alpha_3{}^{-2}] \tag{78}$$

or

$$\alpha_1 = \left(1 - \frac{g_i}{\omega}\right)\frac{\bar{\nu}}{g_r} \quad ; \quad \alpha_2{}^2 = \left(1 - \frac{g_i}{\omega}\right)\left(\frac{\bar{\nu}}{g_r}\right)^2 \quad ; \quad \alpha_3 = \frac{\alpha_2}{\alpha_1} \quad . \tag{79}$$

For $\omega \gg \nu$, $\alpha_1 = \alpha_2 = \alpha_3 = 1$ implies again the ordinary Lorentz equation with $g = \bar{\nu}$. For $\omega = 0$ we obtain Lorentz equation with $\nu_{sc} = \frac{\bar{\nu}}{\alpha_2{}^2\alpha_3{}^2}$ (see Eq. (76)), i.e. $g = \nu_{sc}$.

Whitmer and Hermann have lately[63] introduced ν_{eff} and $(\omega_p)_{eff}$, (for weakly ionized gases only), which are related to our published before[31,32] α's (for any plasma)

$$\left.\begin{aligned} \nu_{eff} &= \frac{1}{\alpha_1}\bar{\nu} \\ (\omega_p)_{eff} &= \alpha_3\omega_p \end{aligned}\right\} \tag{80}$$

These cannot be easily extended in the presence of magnetic fields as our α's can (see Eqs.(143)-(146)) and add by the name, we feel, more confusion to the large number of ill defined effective collision frequencies (see Eq.(49a) and Eq. (185)).

ARGAND TY[illegible] [illegible]AM F[illegible]

Since [illegible]

[illegible] (1)

and

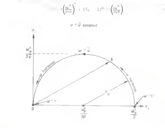
[illegible] (2)

[illegible]

$$\sigma_i^2 + \left(\sigma_r - \frac{\omega_p^2}{2\bar{\nu}}\right)^2 \left(\frac{\omega_p^2}{2\bar{\nu}}\right)^2$$ (3)

[illegible] on of a circle, σ_i represent [illegible] [illegible] sketch) Similarly,

$$-\left(\frac{\omega_p^2}{2\omega}\right)^2 + (\epsilon_r \quad 1)^2 \quad \left(\frac{\omega_p^2}{2\omega\bar{\nu}}\right)$$ [illegible]


$\nu = \bar{\nu}$ constant
σ
$\frac{\omega_p^2}{2\bar{\nu}}$
$\omega = \bar{\nu}$
ω increases
ω increases
σ_o
$\omega = $
0
$\frac{\omega_p^2}{2\bar{\nu}}$
$\frac{\omega_p^2}{\bar{\nu}}$
σ_r

The [illegible] between the real and [illegible]ginary parts [illegible] on the [illegible] to some specific [illegible] angular frequency, [illegible] slope of a line [illegible] from the [illegible] point on a [illegible] (e.g., $\overline{OB}$) is [illegible]. It is [illegible] Eq. (50) or [illegible] diagram, that

$$\left.\begin{aligned} [illegible]\ \sigma_r &\le \frac{a_0^2}{\nu} \\ \text{and} \qquad \sigma\ [illegible] &\frac{a_0^2}{2\nu} \end{aligned}\right\}$$

From Eq. [illegible] for any dependence of [illegible]

[illegible] and [illegible]

and

$$[illegible] + ([illegible] a)^2 \quad a^2 \;,$$

[illegible]

$$[illegible]\ c_1 \qquad a_0 - \frac{\omega_0^2}{2\nu}$$

[illegible] a function of ω, and [illegible] at [illegible]ve, clo[illegible]

[illegible]

[illegible]

[illegible]

$$\text{[illegible]} = \frac{a_0(\mu/\bar{v})a_0(\mu/\bar{v})}{a_0(0)a_0(0)}$$

This [illegible]ation makes it possible [illegible] velocity for any plasma fo[illegible] [illegible] at $\bar{\gamma}_r = 1$, where

$$\text{[illegible]} = \frac{\text{[illegible]}/\bar{v}}{1 + \left(\frac{\mu}{\bar{v}}\right)^2 a_1^{\,2}} \quad , \quad \text{[illegible]} = \frac{\text{[illegible]} + \frac{\nu_{ec}}{\bar{v}}}{1 + \left(\frac{\mu}{\bar{v}}\right)^2 m_1^{\,2}}$$

[illegible] one can simply compute $\bar{\omega}_r$ and $\bar{\omega}_i$ from the a[illegible] relation [illegible] are shown for a few cases in Figs. 1 and 2.

One ca[illegible] [illegible]ow that Eq. 47 can be written as

$$(\text{[illegible]})^2 + (\gamma_r - 1/2)^2 = R^2$$

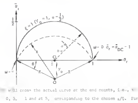
where

$$y_0 = y_0(b) = \frac{\xi(b) - 1}{2b \cdot m_1(b)} \quad , \quad b = \frac{\mu}{\bar{v}} \quad , \quad R = \sqrt{y_0^{\,2} + (1/4)}$$

[illegible] constant this represents an arc of a vertically displaced [illegible] by the amount y_0 and the radius R (see sketch). In general [illegible] is a rather complicated curve, with slightly variable R and [illegible]ly displaced location of the origin O.

[illegible]imately, we can choose any $\mu/\bar{v}$, i.e., y_0 = constant and [illegible] a semicircle with origin displaced vertically by y_0 from the [illegible]

$$\text{[illegible]}_0)^2 + (\gamma - 1/2)^2 = a^2 \quad , \quad R = \sqrt{y_0^{\,2} + (1/2)} \, ,$$


$\bar{\sigma}_i$
$\frac{1}{2}$
$\xi = 1\ (Y_0 = 1,\ a = \frac{1}{2})$
$a = \frac{1}{2}$
$\omega = 0\ \ \bar{\sigma}_r = \bar{\sigma}_{DC} = 1$
$\bar{\sigma}_r$
O_1
R
θ
Y_0
$\omega = \pi$
1
O

[illegible] will cross the actual curve at the end points, i.e., [illegible] 0, $\bar{\sigma}_r$ = 1 and at $\bar{b}_0$ corresponding to the chosen $\omega/\bar{\nu}$. For example, if we choose [illegible] $\omega/\bar{\nu}$ for which $\bar{\sigma}_i$ has the maximum (Pang[38, 38a]), then

$$(\bar{\sigma}_i)_{max} = \bar{\sigma}_i(b_0) \quad ,$$

$$\bar{y}_0 = y_0(b_0) = \frac{\xi(b_0) - 1}{2b_0 + \sigma_i(b_0)}$$

and

$$\omega = \theta(b) = \text{arc tan } 2y$$

We have recomputed [illegible], [illegible], y, etc., and list them in Figs [illegible] and 2 for integral values of n and also for various intermediate n's, using our Table A-1.

A different [illegible] plot of [illegible] vs [illegible] for [illegible] 1 and for a very approximate [illegible] air [illegible] given by Mathews[38b]

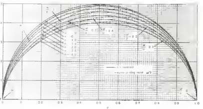


FIGURE 1. The Argand Diagram for the Electric Conductivity of Weakly Ionized Plasma

[illegible] DC [illegible] DC DC ω_p DC [illegible] Av^n

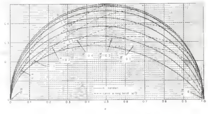


FIGURE 2 The Argand Diagram for the Electric Conductivity of [illegible] Plasma

$\bar{\sigma}$ = σ_r/σ_{DC} [illegible] σ_{DC} σ_i/σ_{DC} [illegible]

[illegible] CONDUC[illegible] [illegible] ATIC MAG[illegible]

[illegible] external[illegible] imp[illegible] assuming at [illegible] -pendent c[illegible] field theo[illegible] we may write [illegible] average velo[illegible] (considering ions as fixed)

$$m_e \vec{v} + m_e \bar{\nu} \vec{v} \; [illegible] \qquad [illegible]$$

which is analogous to Eq. (1[illegible]) w[illegible] field and omitting average sign [illegible] by Appleton[2] and Ha[illegible] tree[2] [illegible] -rent (a.c.) conducti[illegible] [illegible] field. The[illegible] [illegible] at [illegible]

[illegible]

[illegible] will [illegible]

[illegible] -pendent [illegible]

$$\omega_H^e = \frac{-e|H_z^e|}{m_e c} = \frac{|e||H_z^e|}{m_e c} = 1.76 \times 10^7 H_z^e \, \mathrm{sec}^{-1} = 2\pi f_H^{(e)} , \quad (89)$$

where H_z^e is in Gauss. The frequency $f_H^{(e)}$ and ω_H^e are called the electron cyclotron frequency and electron cyclotron angular frequency, respectively. Often it is also called the gyro-frequency. For an earth magnetic field of 0.2 to 0.5 Gauss, $3\times10^6 < \omega_H^e < 9 \times 10^6$ sec^{-1}, i.e. $0.5 < f_H < 1.5$ Mc/s. (Note that the condition $\omega > \omega_H^e$ for an earth field is satisfied for waves of $\lambda < 20$m, i.e., $f > 1.5$ Mc/s.) Then Eq. 88 written out in the components becomes

$$\left.\begin{aligned} m_e\dot{v}_x + m_e\bar{\nu}v_x &= eE_x + \frac{e}{c}v_y H_z \\ m_e\dot{v}_y + m_e\bar{\nu}v_y &= eE_y - \frac{e}{c}v_x H_z \\ m_e\dot{v}_z + m_e\bar{\nu}v_z &= eE_z \end{aligned}\right\} \quad (90)$$

Assuming periodic solutions of the type

$$\vec{E} = \vec{E}_0 e^{-i\omega t} \qquad \text{and} \qquad \vec{v} = \vec{v}_0 e^{-i\omega t} \quad (91)$$

we obtain from the first two equations

$$(\bar{\nu} - i\omega)v_x + \omega_H^e v_y = \frac{e}{m_e} E_x \quad (92)$$

$$-\omega_H^e v_x + (\bar{\nu} - i\omega)v_y = \frac{e}{m_e} E_y \quad (93)$$

The solution of these equations gives

$$v_x = \frac{e}{m_e} \frac{E_x(\bar{\nu} - i\omega) - \omega_H^e E_y}{(\bar{\nu} - i\omega)^2 + (\omega_H^e)^2} \quad (94)$$

$$= \frac{e}{m_e} \frac{[illegible](\ \ - i\omega)\ [illegible]}{(\nu - i\omega) - [illegible]}$$

$$\frac{e}{m_e} \frac{[illegible]_z}{[illegible] - i\omega}$$

[illegible] current density for homoge[illegible]

$$[illegible]\ \vec{E}\ ,$$

[illegible]

$$4\pi J_x\ \ 4\pi N_e e\ [illegible]\ \sigma'_{x}E_x + \sigma'_{y}E_y + \sigma'_{yz}E_z ,$$

$$4\pi J_y - 4\pi N_e e\ v_y\ [illegible]\ E_y + [illegible] + \sigma'_{[illegible]}$$

$$4\pi\ [illegible]\ 4\pi N_e e\ [illegible]$$

[illegible]stituting the expressions for v_x, v_y and v_z we have from Eq[illegible] 95 and 96,

$$4\pi\ [illegible] \frac{[illegible](\ \ i\omega)\ [illegible]}{[illegible]} = AE_x - BE_y\ ,$$

$$4\pi\ [illegible]_y\ [illegible]E_y + B[illegible]$$

$$4\pi N_e e v_z\ [illegible]$$

[illegible]

$$\frac{[illegible]}{(\ \ i\omega)\ (\ \ )}$$

$$[illegible] = \frac{[illegible]}{[illegible]}$$

$$\frac{[illegible]}{(\ \ i\omega)}$$

[illegible]

[illegible]

[illegible] comple[illegible]

[illegible] units, [illegible]

[illegible]

[illegible]

Now we can [illegible] matrix [illegible]

[illegible]

[illegible] ϵ_{xx} [illegible]

[illegible] for ϵ [illegible]

$$\epsilon'_{xx} = \epsilon'_{yy} = 1 + \frac{\text{[illegible]}}{2} \text{[illegible]} \left[\frac{1}{\bar{\nu} - i\Omega_+} + \frac{1}{\bar{\nu} - i\Omega_-}\right] \qquad \text{[illegible]}$$

$$\epsilon'_{xy} = \frac{\sigma'_{xy}}{\epsilon} \text{[illegible]} \left[\frac{1}{\bar{\nu} - i\Omega_+} - \frac{1}{\bar{\nu} - i\Omega_-}\right] \qquad \text{[illegible]}$$

$$\epsilon'_{zz} = 1 + \frac{\text{[illegible]}}{\text{[illegible]}} \text{[illegible]} \frac{1}{\bar{\nu} - i\omega} = 1 - \frac{\omega_p^2}{\omega(\omega + i\bar{\nu})}, \qquad \text{[illegible]}$$

where we define for [illegible] $\Omega_+ = \omega + \omega_H$ and $\Omega_- = \omega - \omega_H$.

Finally,

$$\epsilon_{xx} \; \epsilon \; \epsilon'_{xy} \; \epsilon \qquad \text{[illegible]}$$

Since in many references [illegible] the real part of di[illegible] constant (ϵ_r) and the real [illegible] conductivity (σ_i) we can (see Eq.(113)) from

$$\epsilon'_{k\ell} = \epsilon_{k\ell}^{(r)} + \text{[illegible]} \qquad \text{[illegible]}$$

[illegible] completeness also [illegible]

$$\epsilon_{xx}^{(r)} = \epsilon_{yy}^{(r)} = 1 - \frac{\omega_p^2}{2}\left[\frac{\text{[illegible]}}{\bar{\nu}^2 + \text{[illegible]}} + \frac{\text{[illegible]}}{\text{[illegible]}}\right] \qquad \text{[illegible]}$$

$$\epsilon_{xy}^{(i)} = -\epsilon_{yx}^{(i)} = \frac{\omega_p^2}{2}\left[\frac{1}{\bar{\nu}^2 + \text{[illegible]}} - \frac{1}{\text{[illegible]}}\right] \qquad \text{[illegible]}$$

$$\epsilon_{zz}^{(r)} = 1 - \frac{\omega_p^2}{\bar{\nu}^2 + \omega^2} \qquad \text{[illegible]}$$

and

$$\epsilon^{(i)} \; \epsilon^{(r)} \; \epsilon_{zy}^{(i)} \; \epsilon^{(i)} \qquad \text{[illegible]}$$

For t[illegible] dielectric constant we obtain

$$\epsilon_{k\ell}^{(i)} = \frac{\sigma_{k\ell}^{(r)}}{\omega} , \tag{129}$$

and

$$\text{[illegible]} = \frac{\omega_p^2}{2}\left[\frac{\bar{\nu}}{\bar{\nu}^2+\Omega_+^2} + \frac{\bar{\nu}}{\bar{\nu}^2+\Omega_-^2}\right] \tag{130}$$

$$\text{[illegible]} = \frac{\omega_p^2}{2}\left[\frac{\bar{\nu}}{\bar{\nu}^2+\Omega_+^2} - \frac{\bar{\nu}}{\bar{\nu}^2+\Omega_-^2}\right] \tag{131}$$

$$\text{[illegible]}^{(r)} = \frac{\omega_p^2\,\bar{\nu}}{\bar{\nu}^2+\omega^2} \tag{132}$$

$$\text{[illegible]}^{(r)} \quad \sigma_{yz}^{(r)} = 0 , \tag{133}$$

wh[illegible] [illegible] corresponding relations (k,ℓ = x,y,z) as

$$\text{[illegible]}\ \epsilon_{k\ell}^{(r)} + i(\delta_{k\ell} - \epsilon_{k\ell}^{(r)}) \tag{134}$$

and

$$\text{[illegible]} + \frac{\sigma_{k\ell}}{\text{[illegible]}} \quad \epsilon_{k\ell}^{(r)} + i\,\frac{\sigma_{k\ell}^{(r)}}{\omega} \tag{135}$$

[illegible] imaginary part of electric conductivity [illegible]

$$\left(\delta_{k\ell} - \epsilon_{k\ell}^{(r)}\right)\omega$$

$$\text{[illegible]} = \frac{\omega_p^2}{2}\left[\frac{\Omega_+}{\bar{\nu}^2+\Omega_+^2} + \frac{\text{[illegible]}}{\bar{\nu}^2+\Omega_-^2}\right] \tag{136}$$

$$\text{[illegible]} = \frac{\omega_p^2}{2}\left[\frac{\bar{\nu}}{\bar{\nu}^2+\Omega_+^2} \ \text{[illegible]}\ \frac{\text{[illegible]}}{\bar{\nu}^2+\Omega_-^2}\right] \tag{137}$$

$$\text{[illegible]}^{(i)} = \frac{\omega_p^2\,\omega}{\bar{\nu}^2+\omega^2} \tag{138}$$

13. THE ELECTRIC CONDUCTIVITY OF A WEAKLY IONIZED PLASMA IN THE PRESENCE OF AN EXTERNAL STATIC MAGNETIC FIELD

One can generalize the theory[18,20] of electric conductivity for weakly ionized plasma in the presence of a static magnetic field by allowing the electron collision frequency to be dependant on the electron velocity. One obtains (in our notation, compare with Eq. (34) and Eqs (145) - (149)), (see Appendix H for derivation),

$$\sigma' = \frac{1}{2}\begin{pmatrix} S^+ + S^- & i(S^+ - S^-) & 0 \\ i(S^- - S^+) & S^+ + S^- & 0 \\ 0 & 0 & 2S_0 \end{pmatrix} \qquad (139)$$

i.e.,

$$\left.\begin{aligned} \sigma'_{xx} &= \sigma'_{yy} = \frac{1}{2}(S^+ + S^-) \\ \sigma'_{xy} &= -\sigma'_{yx} = \frac{1}{2}(S^+ - S^-) \\ \sigma' &= S_0 \,, \end{aligned}\right\}$$

where

$$S^+ = -\frac{4\pi\omega_p^2}{3N_e}\int_0^\infty \frac{v^3\frac{\partial f_0^0}{\partial v}\,dv}{\nu - i\Omega_+} = \frac{4}{3\sqrt{\pi}}\,\omega_p^2\int_0^\infty \frac{W\sqrt{W}\,e^{-W}dW}{\nu(W) - i\Omega_+} \qquad \sigma'_{xx} - i\sigma'_{yy} \qquad ([illegible])$$

$$S^- = -\frac{4\pi\omega_p^2}{3N_e}\int_0^\infty \frac{v^3\frac{\partial f_0^0}{\partial v}\,dv}{\nu - i\Omega_-} = \frac{4}{3\sqrt{\pi}}\,\omega_p^2\int_0^\infty \frac{W\sqrt{W}\,e^{-W}dW}{\nu(W) - i\Omega_-} \qquad \sigma'_{xx} + i\sigma' \qquad ([illegible])$$

and

$$[illegible] - \frac{4\pi m_p^2}{3N_e}\int_0^\infty \frac{v^{[illegible]}\,\frac{\partial [illegible]}{\partial v}}{\nu - i\omega} \quad [illegible] \quad \frac{[illegible]}{3\sqrt{\pi}}\,{}' p^2 \int_0^\infty \frac{W\sqrt{W}\,e^{-[illegible]}}{\nu(W) - i\omega} \quad [illegible]$$

[illegible] on the right hand sides we used Maxwellian [illegible] [illegible] [illegible]tion. If $\omega_c{}^* = 0$, $\Omega_+ - \Omega_- = [illegible]$ and [illegible] [illegible] Eq. (54), [illegible] these quantities [illegible] [illegible] that if we define σ_1 and σ_3 for $[illegible]_0$ [illegible] $\sigma_1{}^+$ and $\sigma_3{}^+$ for S^+ and σ_1 and $\sigma_3{}^-$ [illegible] [illegible])

11. GENERALIZATION OF EFFECTIVE ω/ν AND ω_p/ω TO PLASMA IN A STATIC MAGNETIC FIELD

In Eqs. (52) and (53) we defined $(\omega/\nu)_{eff}$, $(\omega_p/\nu)_{eff}$ and $(\omega_p/\omega)_{eff}$ and we tabulated (Table A-1) α_1, α_2 and α_3. Now, we generalize these expressions to (see Eqs. (58),(59),(60),(140),(141),(142)

$$\alpha_1^+ = \frac{(\Omega_+/\nu)_{eff}}{\Omega_+/\nu} \qquad (143)$$

$$\alpha_1^- = \frac{(\Omega_-/\nu)_{eff}}{\Omega_-/\nu} \qquad (144)$$

$$\alpha_3^+ = \frac{(\omega_p/\Omega_+)_{eff}}{\omega_p/\Omega_+} \qquad (145)$$

and

$$\alpha_3^- = \frac{(\omega_p/\Omega_-)_{eff}}{\omega_p/\Omega_-} \ , \qquad (146)$$

where $\Omega_+ = |\omega + \omega_H^*|$ and $\Omega_- = |\omega - \omega_H^*|$. Later we will show that α_1^+, α_1^-, α_3^+ and α_3^- can be read directly from the already tabulated tables for α_1 and α_3. In an analogy to the procedure of defining α_1, α_2 and α_3 in Eqs. (52) and (53) we generalize

$$\epsilon'_{xx} = \epsilon'_{yy} = 1 + \frac{i}{2}\left(\frac{\omega_p}{\omega}\right)_{eff}\left(\frac{\omega_p}{\nu}\right)_{eff}\left[\frac{1}{1-i\left(\frac{\Omega_+}{\nu}\right)_{eff}} + \frac{1}{1-i\left(\frac{\Omega_-}{\nu}\right)_{eff}}\right] \qquad (147)$$

and using our definitions in Eqs (143) to (146), with α_2 and α_3,

$$\epsilon'_{xx} = \epsilon'_{yy} = 1 + i\,\frac{\alpha_2\alpha_3\omega_p{}^2}{2\omega}\left[\frac{1}{\bar{\nu}-i\alpha_1{}^+\Omega_+} + \frac{1}{\bar{\nu}-i\alpha_1{}^-\Omega_-}\right] . \qquad (148)$$

Similarly,

$$\epsilon'_{xy} = -\epsilon'_{yx} = -\frac{\alpha_2\alpha_3\omega_p{}^2}{2\omega}\left[\frac{1}{\bar{\nu}-i\alpha_1{}^+\Omega_+} - \frac{1}{\bar{\nu}-i\alpha_1{}^-\Omega_-}\right] \qquad (149)$$

and

$$\epsilon'_{zz} = 1 + i\,\frac{\alpha_2\alpha_3\omega_p{}^2}{\omega(\bar{\nu}-i\alpha_1\omega)} \qquad (150)$$

[illegible] expressions using Eq. (119)

$$\sigma'_{k\ell} = \omega i(\delta_{k\ell} - \epsilon'_{k\ell})$$

[illegible]

$$\sigma'_{xx} = \sigma'_{yy} = \frac{\alpha_2\alpha_3}{2}(S^+ + S^-) \qquad (151)$$

$$\sigma'_{xy} = -\sigma'_{yx} = i\,\frac{\alpha_2\alpha_3}{2}(S^+ - S^-) \qquad (152)$$

$$\sigma'_{zz} = \frac{\alpha_2\alpha_3\omega_p{}^2}{\bar{\nu}-i\alpha_1\omega} = \alpha_2\alpha_3 S_0 \quad , \qquad (153)$$

where

$$\left.\begin{aligned} S^+ &= \frac{\omega_p{}^2}{\bar{\nu}-i\alpha_1{}^+\Omega_+} \quad , \qquad S^- = \frac{\omega_p{}^2}{\bar{\nu}-i\alpha_1{}^-\Omega_-} \\ \text{and} \quad S_0 &= \frac{\omega_p{}^2}{\bar{\nu}-i\alpha_1\omega} \quad , \end{aligned}\right\} \qquad (154)$$

[illegible]e matrix is

$$\sigma' = \frac{\alpha_2\alpha_3}{2}\begin{Vmatrix} S^+ + S^- & S^+ - S^- & 0 \\ S^- - S^+ & S^+ + S^- & 0 \\ 0 & 0 & 2S_0 \end{Vmatrix}$$

[illegible] ric tensor become

$$\epsilon_{xx}^{(r)} = \epsilon_{yy}^{(r)} = -\frac{\alpha_2\alpha_3\omega_p^2}{2\omega}\left[\frac{(\alpha_1^+)\cdot\Omega_+}{\bar{\nu}^2+(\alpha_1^+)^2\Omega_+^2}+\frac{(\alpha_1^-)\cdot\Omega_-}{\bar{\nu}^2+(\alpha_1^-)^2\Omega_-^2}\right] \quad (155)$$

$$\epsilon_{xy}^{(r)} = -\epsilon_{yx}^{(r)} = -\frac{\alpha_2\alpha_3\omega_p^2}{2}\left[\frac{1}{\bar{\nu}^2+(\alpha_1^+)^2\Omega_+^2}-\frac{1}{\bar{\nu}^2+(\alpha_1^-)^2\Omega_-^2}\right]\frac{\bar{\nu}}{\omega} \quad (156)$$

and

$$\epsilon_{zz}^{(r)} = 1 - \frac{\alpha_1\alpha_2\alpha_3\omega_p^2}{\bar{\nu}^2+\alpha_1^2\omega^2} = 1 - \frac{\alpha_2^2\omega_p^2}{\bar{\nu}^2+\alpha_1^2\omega^2} \quad (157)$$

and the imaginary parts of the dielectric tensor become

$$\epsilon_{xx}^{(i)} = \epsilon_{yy}^{(i)} = \frac{\alpha_2\alpha_3\omega_p^2}{2}\left[\frac{1}{\bar{\nu}^2+(\alpha_1^+)^2\Omega_+^2}+\frac{1}{\bar{\nu}^2+(\alpha_1^-)^2\Omega_-^2}\right]\frac{\bar{\nu}}{\omega} \quad (158)$$

$$\epsilon_{yy}^{(i)} = -\epsilon_{yx}^{(i)} = -\frac{\alpha_2\alpha_3\omega_p^2}{2\omega}\left[\frac{(\alpha_1^+)\cdot\Omega_+}{\bar{\nu}^2+(\alpha_1^+)^2\Omega_+^2}-\frac{(\alpha_1^-)\cdot\Omega_-}{\bar{\nu}^2+(\alpha_1^-)^2\Omega_-^2}\right] \quad (159)$$

and

$$\epsilon_{z}^{(i)} = \frac{\alpha_2\alpha_3\omega_p^2}{\bar{\nu}^2+\alpha_1^2\omega^2}\frac{\bar{\nu}}{\omega} \quad (160)$$

[illegible] need six quantities for a complete determination of real and imaginary parts of ϵ'_{xx}, ϵ'_{xy} and ϵ'_{zz} (e.g., α_1^+, α_1^-, α_3^+, α_3^-, α_1 and [illegible] we know the following equalities

[illegible] $= \left(\frac{\omega_p}{\nu}\right)_{eff}$,

and

$$\boxed{\alpha_2 = \alpha_1^+ = \alpha_1^- ;\ \alpha_3^+ = \alpha_2/\alpha_1^+ ,\ \alpha_3^- = \alpha_2/\alpha_1} \quad (161)$$

[illegible] examples [illegible] weak [illegible] losses, [illegible] suited [illegible] sier is [illegible] of [illegible] energy region of interest), [illegible]

$$[illegible] \quad [illegible]_{en} - NQ_0v^2$$

[illegible] choose only three entries

[illegible]	[illegible]	α_1	α_2^+	α_3
[illegible]	[illegible]	1.[illegible]706	1.4706	0.8873
[illegible]	[illegible]	1.[illegible]660	1.2660	0.9365
[illegible]	[illegible]	1.[illegible]	1.1739	0.9590

[illegible] 1, α = 1.2660 and [illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]87 [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Let $\frac{\Omega}{v}$ [illegible] $\alpha_3^- = 0.8873$.

Thus we found [illegible]

Given:

$$\frac{|\omega + \omega_\mu^*|}{v} = \text{[illegible]}, \quad \frac{\omega}{v} = 1.00 \text{ and } \frac{|\omega - \omega_\mu^*|}{v} \text{ [illegible]}$$

(e.g., $\frac{\omega}{v} = 1.0$, $\frac{\omega_\mu^*}{v} = 0.5$), then for $\nu_{en} = NQ_0 v^2$ we obtain

$$\alpha_1 = 1.3518 \qquad \alpha_1^+ = 1.224 \qquad \alpha_1^- = 1.6575$$

$$\alpha_3 = 0.9365 \qquad \alpha_3^+ = 0.5590 \qquad \alpha_3^- = 0.8873$$

and $\alpha_2 = \alpha_2^+ = \alpha_2^- = 1.2660$.

In order to clarify the use of Table A-1, we decided to head the α_1 or α_1^+ or α_1^- as a_1, α_2, α_2^+ or α_2^- as a_2 and α_3 α_3^+ or α_3^- as a_3 (see Table 1-A and the footnote).

Example of Special Cases

1. If $u = \omega_H^e = 0$ (D.C. Case), [illegible]

$$\alpha_1 = \alpha_1^+ = \alpha_1^- \quad , \quad [illegible]$$

$$\sigma' \quad \sigma'_{zz} = \sigma'_{xx} = \sigma'_{yy} = \frac{\omega_p^2}{\nu_{ec}} \quad [illegible] \quad \frac{\bar{\nu}}{\alpha_1 \alpha_3^2}$$

$$\sigma'_{xy} = \sigma'_{yx} = 0$$

2. If $\omega = 0$, $\omega_H^e \neq 0$ (D.C. Case with a [illegible] field)

$$\Omega_+ = \Omega_- = \omega_H^e$$

$$\alpha_1^+ = \alpha_1^- \quad ; \quad \alpha_2 [illegible]$$

$$S_0 = \frac{\alpha_1 \alpha_3 \omega_p^2}{\bar{\nu}}$$

$$\sigma'_{xx} = \sigma'_{yy} = \frac{\alpha_1^+ \alpha_3^+ \omega_p^2}{\nu - i \alpha_1^+ \omega_H^e}$$

$$\sigma_{xy} = \sigma'_{yx} = 0$$

$$[illegible] = \frac{1}{b} \quad , \quad \nu_{ec} = \frac{\bar{\nu}}{\alpha_1 \alpha_3^2} = \frac{[illegible]}{\alpha_2 \alpha_1}$$

3. If [illegible] $\omega_H^e \neq 0$

$$[illegible] \quad ; \quad \Omega_- = 0 \quad ;$$

4. If [illegible] ω_H^e [illegible] Ω [illegible] ω

$$[illegible] \quad \alpha_3 = \alpha_3^+ = \alpha_3^-$$

$$[illegible] \sigma'_{zz} = \frac{\alpha_1 \alpha_3^2 \omega_p^2}{\bar{\nu} - i \alpha_3 [illegible]}$$

$$[illegible] = \sigma'_{yx} = 0$$

If u >> v

a. [illegible]

[illegible]

If $u_*^e >> 1$

b [illegible]

Comparison with Other Notations

Shkarofsky[25] defined six other [illegible] g_- and h_- by

$$\left.\begin{aligned}
\sigma'_{xx} &= \frac{\omega_p^2}{2}\left(\frac{1}{\bar{\nu}g_+ - i\Omega_- h_+} + \frac{1}{\bar{\nu}g_- - i\Omega_+ h_-}\right)\\
\sigma'_{xy} &= i\,\frac{\omega_p^2}{2}\left(-\frac{1}{\bar{\nu}g_+ - i\Omega_- h_+} + \frac{1}{\bar{\nu}g_- - i\Omega_+ h_-}\right)\\
\sigma'_{zz} &= \frac{\omega_p^2}{\bar{\nu}g_0 - i\omega h_0}\ .
\end{aligned}\right\} \quad (162)$$

Comparing with our notation one can show that

$$\left.\begin{aligned}
&\alpha_1^- = h_+/g_+ \ , \quad \alpha_1^+ = h_-/g_- \ , \quad \alpha_1 = h_0/g_0\\
&\alpha_3^- = \frac{\sqrt{h_0}}{h_+} \ , \quad \alpha_3^+ = \frac{\sqrt{h_0}}{h_-} \ , \quad \alpha_3 = \frac{1}{\sqrt{h_0}}
\end{aligned}\right\} \quad (163)$$

and

$$\alpha_2 = \alpha_1\alpha_3 = \frac{\sqrt{h_0}}{g_0}$$

Ginzburg[6] uses six other quantities, $K_\varepsilon = K_\varepsilon(\Omega_+/\bar{\nu})$, $K_\varepsilon(\Omega_-/\bar{\nu})$, $K_\varepsilon(\omega/\bar{\nu})$, $K_\sigma(\Omega_+/\bar{\nu})$, $K_\sigma(\Omega_-/\bar{\nu})$ and $K_\sigma(\omega/\bar{\nu})$ defined by [$K_\varepsilon(\Omega_+/\bar{\nu})$ means K_ε as a function of $\Omega_+/\bar{\nu}$, etc.]

$$\varepsilon_{xx}^{(r)} = 1 - \frac{\omega_p^2}{2\omega}\left[\frac{\Omega_+ K_\varepsilon(\Omega_+/\bar{\nu})}{\bar{\nu}^2 + \Omega_+^2} + \frac{\Omega_- K_\varepsilon(\Omega_-/\bar{\nu})}{\bar{\nu}^2 + \Omega_-^2}\right] \quad (164\text{ a})$$

$$\varepsilon_{xy}^{(r)} = -\frac{\omega_p^2}{2}\left[\frac{K_\varepsilon(\Omega_+/\bar{\nu})}{\bar{\nu}^2 + \Omega_+^2} - \frac{K_\varepsilon(\Omega_-/\bar{\nu})}{\bar{\nu}^2 + \Omega_-^2}\right]\frac{\bar{\nu}}{\omega} \quad (164\text{b})$$

$$\varepsilon_{zz}^{(r)} = 1 - \frac{\omega_p^2}{\bar{\nu}^2 + \omega^2}\,K_\varepsilon(\omega/\bar{\nu}) \quad (164\text{c})$$

$$\sigma_{xx}^{(r)} = \frac{\omega_p^2}{2}\left[\frac{K_\sigma(\Omega_+/\bar{\nu})}{\bar{\nu}^2 + \Omega_+^2} + \frac{K_\sigma(\Omega_-/\bar{\nu})}{\bar{\nu}^2 + \Omega_-^2}\right]\bar{\nu} \tag{164d}$$

$$\sigma_{xy}^{(r)} = -\frac{\omega_p^2}{2}\left[\frac{K_\sigma(\Omega_+/\bar{\nu})}{\bar{\nu}^2 + \Omega_+^2} - \frac{K_\sigma(\Omega_-/\bar{\nu})}{\bar{\nu}^2 + \Omega_-^2}\right]\bar{\nu} \tag{164e}$$

$$\sigma_{zz}^{(r)} = \frac{\omega_p^2\bar{\nu}}{\bar{\nu}^2 + \omega^2}\, K_\sigma(\omega/\bar{\nu}) \quad , \tag{164f}$$

however his Eq. (10.32) contains many misprints.

For $\omega/\bar{\nu} \to \infty$ and $|\omega \pm \omega_c^*|/\bar{\nu} \to \infty$, $\alpha_1(\infty)$, $\alpha_3(\infty) \to 1$ and also α_1^+, α_1^-, α_3^+, α_3^-, h_0, h_+, h_-, g_0, g_+, g_-, $K_\sigma(\infty)$ and $K_\epsilon(\infty) \to 1$

ELECTRON-NEUTRAL ELASTIC CROSS SECTIONS

If the differential cross section for the electron scattered elastically from the i-th neutral atom or molecule through an angle θ and with relative velocity v is $\sigma^{(i)}(v,\theta)$, then the scattering cross section for the momentum transfer, (sometimes referred to as the diffusion cross section), in the center of the mass system, is[28]

$$Q_m^{(i)}(v) = 2\pi \int_0^{\pi} \sigma^{(i)}(v,\theta)(1 - \cos\theta) \sin\theta \, d\theta, \qquad (165)$$

This cross section is needed in the calculation of the electron neutral collision frequency.

The total cross section, which is usually the one directly measured is (in the absence of inelastic collisions)

$$Q_t^{(i)} = 2\pi \int_0^{\pi} \sigma^{(i)}(v,\theta) \sin\theta \, d\theta \qquad (166)$$

For sufficiently slow electrons scattered from a neutral atom or molecule, the scattering is usually independent of an angle (e.g., for noble gases[9] for $E \leq 5$ ev, but probably not for sodium which may have a high resonance) and there is little difference between cross section for the momentum transfer and the total cross section, i.e.

$$Q_m^{(i)} \simeq Q_t^{(i)}$$

Theoretically one usually obtains phase shifts $\eta_\ell^{(i)}$ from quantum mechanics. Then[28]

$$Q_t^{(i)} = \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) \sin^2 \eta_\ell^{(i)} \qquad (167)$$

where

$$k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{mv}{\hbar} ,$$

m is the reduced mass ($m \approx m_e$) and $\hbar$ is the Planck constant divided by 2π. One can also show that[29,30]

$$Q_m^{(i)} = \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (\ell + 1) \sin^2(\eta_\ell^{(i)} - \eta_{\ell+1}^{(i)}) \qquad (168)$$

At low energies only a few phase shifts (i.e., mostly with $\ell = 0$ (s-wave), sometimes also with $\ell = 1$ (p-wave), and $\ell = 2$ (d-wave) and rarely higher) contribute to the cross sections.* Usually $Q_t^{(i)}$ or $\sigma^{(i)}$ are reported from experimental scattering data (e.g., using molecular beams) as $Q = Q(E_{ev})$, where E_{ev} is the energy of electrons in electron volts. For electron-atom or electron-molecule cross sections the conversion relation (for nonrelativistic energies) is

$$v_{cm/sec} = 5.932 \times 10^7 \sqrt{E_{ev}} .$$

Often the cross-sections are reported in πa_o^2, where

$$Q(\text{in cm}^2) = 8.806 \times 10^{-17}\ Q\ (\text{in units of } \pi a_o^2) ,$$

---

*Note that if $\ell = 0$ only, $Q_t^{(i)} = Q_m^{(i)}$

Frequently experimental data are given for collision probability P_c, i.e., fraction of electrons scattered out of a beam per 1 centimeter of path in a gas pressure of 1 millimeter of mercury at 0^oC,

$$Q \text{ (in cm}^2) = 0.02823 \times 10^{-15} \quad P_c \text{ (in } \frac{\text{cm}}{\text{mmHg}}),$$

or

$$Q \text{ (in } \pi a_o{}^2) = 0.322 \cdot P_c \text{ (in } \frac{\text{cm}}{\text{mmHg}}) \quad .$$

Also 1 Rydberg - 1 Hartree = 13.605 ev and

$$E_{ev} = 13.605(ka_o)^2 = 13.605\ k_a{}^2 - 27.210\ E_a = 13.605E_{Rydbergs},$$

where k_a and E_a are the wave number and energy in atomic units respectively. Further,

$$ka_o = 0.2711\sqrt{E_{ev}}$$

The approximate range of energies at which cross-sections are needed (for a given temperature) for calculation of collision frequencies are given in Table 5. Thus, we see that for $T = 300^oK$ we need mostly cross-sections at very low energies, i.e., at $7 \times 10^{-3} \lesssim E \lesssim 0.3$ ev and at 6000^oK we need cross-sections at higher energies $0.1 \lesssim E \lesssim 6.5$ ev. Actually one needs in evaluating the integral for collision frequency, also cross sections at lower and higher energies than indicated, however, the most significant contribution to the integral will come from this range and from nearby energies at which cross sections have resonances.

Often one uses integration over impact parameters ρ (see Chap 1, Ref 60), then one can show[29] that

$$\rho d\rho = \sigma(\theta,v)\sin\theta d\theta$$

and

$$Q_m^{(1)}(v) = 2\pi\int_0^\infty (1 - \cos\theta)\rho d\rho \qquad (10')$$

In the Appendix G we have reviewed and listed various experimental cross sections.

13. ELECTRON NEUTRAL CROSS-SECTIONS FOR POLAR MOLECULES

For polar molecules, i.e., for molecules possessing permanent electric dipole moment, Altshuler[47] derived momentum transfer cross-section (using Born approximation)

$$Q_m = \frac{8\pi}{3}\left(\frac{\mu_e e}{\hbar v}\right)^2 \quad , \qquad (169a)$$

or

$$Q_m = 5.60\,\frac{\mu_e{}^2}{E_{ev}} \text{ in } \pi a_0{}^2 = 4.94 \times 10^{-16}\,\frac{\mu_e{}^2}{E_{ev}}\ \text{cm}^2 \ , \qquad (169b)$$

where μ_e is the permanent electric dipole moment in Debye units[48-52] (1 debye = 10^{-18} esu) and E_{ev} is the electron energy in electron volts.

For atoms, homonuclear molecules (e.g., O_2, N_2, Cl_2) or linear symmetric molecules (e.g., CO_2: O = C = O) and for molecules with a center of symmetry (e.g., C_6H_6) the electric dipole moment is zero and the above formula is not applicable (this includes the following molecules: SO_3, SF_6, S_2F_{10}, H_2O_2, CCl_4, CO_2, CH_4, C_2N_2, C_2H_2, C_2H_4, C_6H_6, etc.)

In Table 1 are given empirically determined electric dipole moments for a few selected molecules. For complex molecules one can obtain μ_e by vector addition of dipole moments of single bonds. Thus, Eucken[53] found μ_e for each bond

TABLE 1

THE ELECTRIC DIPOLE MOMENTS OF SELECTED MOLECULES IN A GASEOUS STATE*

	μ_e (Debyes)
NO	0.0 - 0.16
CO	0.10
HCl	1.07
HLi	5.88 - 5.99
OH	1.65 ± 0.25
NaCl	8.5 ± 0.4
CsF	7.3 - 7.58
CsI	10.2
CsCl	~ 10.5
H_2O	1.82
D_2O	1.85 - 2.04
H_2S	0.897 - 1.101
NH_3	1.47
CHN	2.91
CH_4O	1.70
CH_5N	1.24
CH_3NO_2	3.44
$C_6H_5NO_2$	4.2 ± 0.5

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*From Ref. 48-52

	H-C	H-O	C=O	C-O	C-Cl
μ_e, Debyes	0.4	1.6	2.3	0.7	1.5

	C-OH	$\overset{O}{\text{C-C-OH}}$	C-NO_2
μ_e, Debyes	1.6	1.0	3.7

The accuracy of the so determined electron molecule cross-section for momentum transfer depends on the validity of the Born approximation.

Since the derived $Q_m \sim E^{-1}$ or $\sim 1/v^2$ and the electron neutral collision frequency $\nu \sim 1/v$, we can use Table A-1 for $n = -1$ ($s = -1$) and for pure gas consisting of a single species of polar molecules we obtain the effective corrections to the electric conductivity, i.e., α_1, α_2 and α_3, $\bar{\nu}_{DC} = 0.8836\ \bar{\nu}$ and from Table B-1 for $n = -1$ ($s = -1$) one can obtain the contribution of a given i-th polar molecule to the total electron neutral collision frequency $\bar{\nu}_{e,n}(\omega >> \nu)$, i.e., from Eq. 308

$$\bar{\nu}_{e,n}^{(i)} = D_{-1}^{(i)} q_{-1}^{(i)} N_{cm^{-3}}^{(i)} \quad , \qquad (169c)$$

where

$$q_{-1}^{(i)} = 4.54 \times 10^{-16}\ \mu_e^{\ 2} \qquad (169d)$$

Thus, for example, for a weakly ionized H_2O vapor at 300°K the electron-neutral collision frequency ($\omega >> \bar{\nu}$) is

$$\bar{\nu}_{e,n}^{(H_2O)} = 2.7752 \times 10^8\ (4.54 \times 10^{-16})\ (1.82)^2\ N_{cm^{-3}}^{(H_2O)}$$

$$= 4.54 \times 10^{-7}\ N_{cm^{-3}}^{(H_2O)}\ ,\ sec^{-1}$$

ELECTRON COLLISION FREQUENCY WITH
NEUTRAL ATOMS AND MOLECULES

The electron neutral collision frequency, $\nu_{e,n}^{(i)}$, i.e., the number of collisions per second an electron makes with i-th type neutral atoms or molecules, is*

$$\nu_{e,n}^{(i)}(v) = N_i Q_m^{(i)}(v) \cdot v \quad ,$$

where N_i is the number of i-th neutral atoms or molecules per cubic centimeter, $Q_m^{(i)}$ is the momentum transfer elastic cross section (in cm^2) for electron with i-th atom or molecule and v is the electron velocity in cm/sec.

The total collision frequency (for momentum transfer) of an electron with neutral atoms and molecules is

$$\nu_{e,n}(v) = \sum_i N_i Q_m^{(i)}(v) \cdot v \quad . \tag{170}$$

In principle for a gas (at high temperature and up to a moderate density) consisting of atoms and molecules in various excited states, one should consider each state of an atom and molecule as a separate i-th species, with the corresponding electron elastic cross section, $Q_m^{(i)}$. However, at present we do not know much about momentum transfer elastic cross sections for electrons with atoms and molecules in various excited states.

---

* We are neglecting inelastic collisions, i.e., resulting in the ionization, recombination, excitation and radiation.

For $\omega >> \nu$, the average electron neutral collision frequency is (see Eq. (49)) for electrons with i species for a Maxwellian velocity distribution

$$\bar{\nu}_{e,n}^{(i)} = \frac{8}{3\sqrt{\pi}}\left(\frac{m_e}{2kT_e}\right)^{5/2} \int_0^\infty \nu_{e,n}^{(i)}(v) \cdot v^4 e^{-\frac{1}{2}\frac{mv^2}{kT_e}} dv$$

$$= \frac{8}{3\sqrt{\pi}}\left(\frac{m_e}{2kT_e}\right)^{5/2} N_i \int_0^\infty Q_m^{(i)}(v) \cdot v^5 e^{-\frac{1}{2}\frac{mv^2}{kT_e}} dv$$

$$= \frac{4}{3\sqrt{\pi}}\left(\frac{2kT_e}{m_e}\right)^{1/2} N_i \int_0^\infty Q_m^{(i)}(W) W^2 e^{-W} dW ,$$

$$\bar{\nu}_{e,n} = \sum_i \bar{\nu}_{e,n}^{(i)} \tag{171}$$

It is important to point out that not the collision frequency ν, but $\nu \cdot v^2$ must be averaged over velocity distribution of electrons (e.g., Maxwellian). This often is a source of a considerable error and an origin of an unnecessary variety of "effective" collision frequencies (Eq.(185),etc.). The case of $\bar{\nu}$ (called by Ginzburg[6], ν_{eff}) for $\omega >> \nu$ is to be preferred since it forces the corrections α_1, α_2 and α_3 in the electric conductivity to be unity for $\omega >> \nu$, and $\bar{\nu}$ can be compared directly with the measured value for $\omega >> \nu$.

Collision Frequency for Rigid (Elastic, Spherical) Neutral Atoms and Molecules*

For an electron scattered from a neutral particle (N particles in 1 cm^3 of a weakly ionized plasma) having radius r_o (collision distance) from the kinetic theory of gases

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* See case of n = 1, Table A-1

$$\sigma(v,\theta) = \frac{r_o^2}{4} \quad , \quad Q_o = \pi r_o^2 \quad , \tag{172}$$

where Q_o is the constant cross section and

$$\nu_{e,n} = N\cdot(\pi r_o^2)\cdot v \quad . \tag{173}$$

Averaging over Maxwellian distribution (with constant πr_o^2)

$$<\nu_{e,n}> = N(\pi r_o^2)\,\bar{v}, \ \bar{v} = \sqrt{\frac{8kT_e}{\pi m_e}} \tag{174}$$

For the case of $\omega >> \nu$, from Eq. (48) for Maxwellian distribution for a weakly ionized plasma, we obtain the proper average (see next section)

$$\bar{\nu}_{e,n} \cong \frac{\overline{\nu v^2}}{\overline{v^2}} = \frac{4}{3}N(\pi r_o^2)\,\bar{v} = \frac{4}{3}<\nu_{e,n}> . \tag{175}$$

This is the same expression as one would obtain from the kinetic theory for rigid spheres and which was often used for the ionosphere.[6,30]

If Q is the electron-neutral cross-section for the momentum transfer and if at low energies $Q \sim Q_o$ = constant, then from Eq. (49)

$$\nu_{e,n} = NQ_o v \quad , \quad \bar{\nu}_{e,n} = \frac{4}{3}NQ_o\bar{v} = \frac{8}{3\sqrt{\pi}}NQ_o\sqrt{\frac{2kT_e}{m_e}} \tag{176}$$

Various Definitions of Average Collision Frequency

Often in the literature is used (for a general velocity dependent cross section, it has no significance) the collision frequency taken at the energy equal to kT_e, i.e., at

$$E_* = kT_e = \frac{1}{2} m_e v_*^2 \quad , \quad \nu(E_*) \quad \nu^* \quad ,$$

at which the electron neutral elastic cross-section for the momentum transfer has the value

$$Q_m^* = Q(E_*) \quad , \tag{177}$$

For a simple energy dependence of the cross-section of the j-th neutral species given by

$$Q_m^{(j)} = q_s^{(j)} E_{ev}^{\,s} = q_n^{(j)} v^{n-1} \quad , \quad s = \frac{n-1}{2} \quad , \tag{178}$$

$$Q_m^{*(j)} \quad q_s^{(j)} E_{*,ev}^{\,s} = q_s^{(j)} \cdot \left(\frac{T_e(^\circ K)}{11605}\right)^s \tag{178a}$$

and

$$\nu_{e,n}^{*(j)} = N_j \left(\frac{2kT_e}{m_e}\right)^{1/2} q_s^{(j)} \left(\frac{T_e}{11605}\right)^s \tag{179}$$

From Eq. (178), Eq. (49)

$$\bar{\nu}_{e,n}^{(j)} = \frac{4}{3\sqrt{\pi}} \Gamma(s+3) \, \nu_{e,n}^{*(j)} \quad , \tag{180}$$

where the gamma function

$$\Gamma(m+1) = \int_0^\infty W^m e^{-W} \, dW \quad . \tag{181}$$

Often we will use the following values of the gamma function[33]

$$\Gamma(m+1) = m! \text{ , for } m = 0, 1, 2, 3, \ldots$$

$$\left.\begin{array}{ll} \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \quad ; & \Gamma\left(\frac{5}{2}\right) = \frac{3}{4}\sqrt{\pi} \quad , \\ \Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{\pi} \quad , & \Gamma\left(\frac{7}{2}\right) = \frac{15}{8}\sqrt{\pi} \quad , \end{array}\right\} \qquad (182)$$

$$\Gamma(n+\tfrac{1}{2}) = \frac{1.3 \cdot 5 \cdots (2n-1)}{2^n}\sqrt{\pi} \ .$$

Another average collision frequency often used, and again of no significance for the general velocity dependent cross-section, is the collision frequency averaged over Maxwellian distribution,

$$<\nu> = \left(\frac{m_e}{2\pi kT_e}\right)^{3/2} \int_0^\infty \nu(v)\, e^{-\frac{1}{2}\frac{m_e v^2}{kT_e}}\, 4\pi v^2 dv \ , \qquad (183)$$

instead of the correct average $\bar{\nu}$ (see Eq. 61).

For $\omega >> \nu$ only $\bar{\nu}$ can be directly compared with experimental frequency for weakly and for fully ionized plasma. (For $\omega << \nu$, only ν_{DC} can be uniquely defined and experimentally deduced, see Eq. 51). Only for a simple energy dependence of the cross-section ν^* and $<\nu>$ are different by a constant factor from $\bar{\nu}$ (or ν_{DC}), i.e ,

$$\frac{\bar{\nu}_{e,n}^{(j)}}{<\nu_{e,n}^{(j)}>} = \frac{2}{3}\,\frac{\Gamma\left(\frac{n+5}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)} \quad , \qquad (184)$$

and from Eq. (180) and Eq. (184) (omitting j-th superscript) we obtain

$Q \sim E^{-2}$, [illegible] $\sim v^{-3}$, $\bar{\nu}_{e,n} = \frac{4}{3\sqrt{\pi}} \nu^{*}_{e,n}$. (185a)

$Q \sim E^{-\nu/2}$, [illegible] $^{-2}$, $\bar{\nu}_{e,n}$ $\frac{2}{3}$ $\nu^{*}_{e,n}$ $\frac{1}{3}$ [illegible] $_{e,n}$ (185b)

[illegible] $^{-1}$, Q [illegible] $^{-1}$ [illegible] $_{e,n} \sim \frac{4}{3\sqrt{\pi}} \nu^{*}_{e,n} - \frac{2}{3}$ [illegible] $_{e,n}$ (185c)

[illegible] ^{1} [illegible] $^{-1}$ [illegible] onst , $\bar{\nu}_{e,n}$ $\nu^{*}_{e,n} =$ [illegible] $_{e,n}$ (185d)

Q constant [illegible] , [illegible] $\frac{8}{3\sqrt{\pi}} \nu^{*}_{e,n} - \frac{4}{3} \langle \nu_{e,n} \rangle$ (185e)

$Q \sim E^{1/2}$, $Q \sim v$, $\sim$ [illegible] $\bar{\nu}_{e,n} = \frac{5}{2} \nu^{*}_{e,n}$ $\frac{5}{3} \langle \nu_{e,n} \rangle$ (185f)

$Q \sim E$, $Q \sim v^{2}$, [illegible] $\sim$ [illegible] $_{e,n}$ $\frac{8}{\sqrt{\pi}}$ [illegible] $^{*}_{e,n} - 2$ [illegible] $_{e,n} \rangle$ (185g)

The ratio of ν_{DC} to $\bar{\nu}$ for various n is given in Table 3 and in Figure 6.

15. CORRECTIONS TO ELECTRIC CONDUCTIVITY FOR A ONE NEUTRAL COMPONENT WEAKLY IONIZED PLASMA WITH GIVEN $Q = Q_o v^{n-1}$

Let the electron neutral cross-section for the momentum transfer for a given one component gas be given by

$$Q = Q_o v^{n-1} \quad , \tag{186}$$

where Q_o is a constant. This dependence could be obtained from experimental cross-sections in the low energy range ($E \lesssim 1$ ev), since the exponential weighting factor in Eq. (35) gives only significance to the low velocity contributions.

We have

$$\nu_{e,n} = NQv = NQ_o v^n = NQ_o \left(\frac{2kT_e}{m_e}\right)^{n/2} w^{n/2} \quad , \tag{187}$$

and for [illegible] >> [illegible], the average effective collision frequency for a weakly ionized plasma is from Eq. (61) for Maxwellian distribution

$$\nu_{e,n} = \frac{4}{3\sqrt{\pi}} NQ_o \cdot \left(\frac{2kT_e}{m_e}\right)^{n/2} \Gamma\left(\frac{n+5}{2}\right) \tag{188}$$

$$\frac{v_{e,n}}{\bar{v}_{e,n}} \quad [illegible]$$

$$c_n = \frac{4}{3\sqrt{\pi}}\,\Gamma\left(\frac{n+5}{2}\right)$$

$$c_{-1} = \frac{4}{3\sqrt{\pi}} \quad , \quad c_{-2} = \frac{2}{3} \quad , \quad c_{-3} = \frac{4}{\sqrt{\pi}}$$

$$c_1 = \frac{8}{3\sqrt{\pi}} \quad , \quad c_2 = \frac{5}{2} \quad , \quad c_3 = \frac{4}{\sqrt{\pi}}$$

1 Furthermore, let

$$b = \frac{1}{\bar{v}_{e,n}} \quad \left(\text{or } b = \frac{|\omega k m_e^*|}{\bar{v}_{e,n}}\text{, see Eq. (88), etc.}\right),$$

from Eq. (46) and Eq. (47) we obtain for Maxwellian distribution

$$I_1 = \frac{4}{3\sqrt{\pi}} c_n \int_0^\infty \frac{W^{\frac{n+3}{2}} e^{-W} dW}{W^n + c_n^2 b^2}$$

$$= \frac{1}{\sqrt{\pi}} c_n \int_0 \frac{W^{3/2} e^{-W} dW}{W^n + c_n^2 b^2}$$

The corrections a_1, a_2 and a_3 (or $a_1^{\pm}$ and $a_3^{\pm}$) can then be obtained from Eq. (54) through Eq. (58).

For integral values of n (n = -3, -2, -1, 1, 2, 3) and and $b = \frac{|\omega \pm \omega_c^*|}{\nu}$, one can use functions discussed in detail and tabulated by Dingle et al[34] For the case of $n = \pm 2$ I_1 and I_2 are related to the sine and cosine integrals (or Fresnel integrals) and for $n = \pm 1$ to the generalized exponential integrals. For the case of $n = \pm 1$ and $n = 2$, Molmud[35] rederived the same relations. Using Dingle et al.[34] tables, Shkarofsky[25] tabulated functions g_σ and h_σ (see Eqs (72) and (73)) for a few special values of b

In Table A-2 and Table A-1 and in Fig. 3-5, we list our results for n = -3, -2.5, -2, -1.5, -1.0, -0.5, 1, 1.5, 2, 2 5 and n = 3 obtained by numerical integration For the integral values of n, our solutions agree with the values obtained using Dingle et al.[34] tables.

In the next section we will discuss in detail the cases of integral values of n, the connection to Dingle et al. functions and the asymptotic relations. Although a direct numerical integration of Eq (152) and Eq (153) is very simple and straightforward (see Eq. (268)) and which we used and recommend for any other computation, we are compelled to list in the next section the relations for the integral values of n because of a large number of misprints which appeared in the literature while quoting these relations[6,14,38] and insufficient number of terms in the asymptotic formulae to guarantee convergence even to the first digit.[38]

Case of $n = -3$, $Q = Q_0 v^{-4}$, $\nu = NQ_0 v^{-3}$

$$I_1 = \frac{4}{3\sqrt{\pi}} c_{-3} \int_0^\infty \frac{W^3 e^{-W} dW}{1+c_{-3}^2 b^2 W^3} = \frac{32}{3\pi} \mathfrak{C}_3\left(\frac{16}{9\pi} b^2\right) \tag{194}$$

$$I_2 = \frac{4}{3\sqrt{\pi}} c_{-3}^2 \int_0^\infty \frac{W^{9/2} e^{-W} dW}{1+c_{-3}^2 b^2 W^2} = \frac{70}{\pi} \mathfrak{C}_{4.5}\left(\frac{16}{9\pi} b^2\right) \tag{195}$$

$$\bar{\nu}_{e,n} = \frac{4}{3\sqrt{\pi}} NQ_0 \left(\frac{m_e}{2KT}\right)^{3/2} \tag{196}$$

From Dingle et al.[34] for $b = 0$

$$\mathfrak{C}_3(0) = \mathfrak{C}_{9/2}(0) = 1 \tag{197}$$

and from Eq. (58),

$$a_1 = \frac{105}{16} \quad , \quad a_2 = \sqrt{\frac{70}{\pi}} \tag{198}$$

Fang[38] gives asymptotic expressions (article contains many misprints) for $\mathfrak{C}_p$ derived from Dingle et al.[34] who themselves resorted mostly to numerical integration in preparation of their tables of $\mathfrak{C}_p$. Sengupta[14] erroneously related these to K_ν.

For electron-ion collisions, the Coulomb cross-section has approximately v^{-4} dependence, however this case should not be used for fully ionized plasma for $\omega \lesssim \nu$ because the above theory for weakly ionized plasma neglects electron-electron collision which are important for $\omega \lesssim \nu$. (Eq. (196) can be approximately used for $\omega >> \nu$ if the expression in Eq. (196) is multiplied by a logarithmic term, see Ch. II, Ref. 60 and for high z plasma for any ω/ν ratio, see Ch. II, Ref. 60 and Ref. 61.

Case of $n = -2$, $Q = Q_o v^{-3}$, $\nu = NQ_o v^{-2}$

$$I_1 = \frac{4\, c_{-2}^{-1}}{3b^2\sqrt{\pi}} \int_0^{\infty} \frac{W^{5/2} e^{-W} dW}{W^2 + b^{-2} c_{-2}^{-2}} = \frac{15}{4b^2}\, \mathfrak{C}_{\frac{5}{2}}\left(\frac{3}{2b}\right) \tag{199}$$

$$I_2 = \frac{4}{3b^2\sqrt{\pi}} \int_0^{\infty} \frac{W^{7/2} e^{-W} dW}{W^2 + b^{-2} c_{-2}^{-2}} = \frac{35}{4b^2}\, \mathfrak{C}_{\frac{7}{2}}\left(\frac{3}{2b}\right) , \tag{200}$$

$$\bar{\nu}_{e,n} = \frac{2}{3} NQ_o \cdot \frac{m_e}{2kT_e} . \tag{201}$$

The $\mathfrak{C}_p(x)$ function is related to the Fresnel integrals[34]

$$\begin{aligned} C(u) &= \frac{1}{\sqrt{2\pi}} \int_0^u \frac{\sin t}{\sqrt{t}} = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt = C(x) \\ S(u) &= \frac{1}{\sqrt{2\pi}} \int_0^u \frac{\cos t}{\sqrt{t}}\, dt = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt = S(x) , \end{aligned} \tag{202}$$

where $x = \frac{\pi}{2} u^2$. Abreviating these through the sine and cosine integrals

$$\begin{aligned} Ci_{\frac{1}{2}}(x) &= \sqrt{\frac{2\pi}{x}}\left[\frac{1}{2} - C(x)\right] \\ Si_{\frac{1}{2}}(u) &= \sqrt{\frac{2\pi}{x}}\left[\frac{1}{2} - S(x)\right] \end{aligned} \tag{203}$$

Dingle, et al.[34] obtained

$$\begin{aligned} \mathfrak{C}_{-\frac{1}{2}}(x) &= \frac{1}{x}\left[Si_{\frac{1}{2}}(x) \cos x - Ci_{\frac{1}{2}}(x) \sin x\right] \\ \mathfrak{C}_{\frac{1}{2}}(x) &= 2\left[Si_{\frac{1}{2}}(x) \sin x + Ci_{\frac{1}{2}}(x) \cos x\right] \end{aligned} \tag{204}$$

and from the recurrence relation[34]

$$\mathfrak{C}_{\frac{5}{2}}(x) = \frac{4}{15}\left[1 - x^2\,\mathfrak{C}_{\frac{1}{2}}(x)\right]$$
$$\mathfrak{C}_{\frac{7}{2}}(x) = \frac{4}{35}\left[1 - \frac{4}{3}x^2 + \frac{4}{3}x^4\,\mathfrak{C}_{\frac{1}{2}}(x)\right] \qquad (205)$$

and (needed for n = 2 case)

$$\mathfrak{C}_{\frac{3}{2}}(x) = \frac{4}{3}\left[1 - x^2\,\mathfrak{C}_{-\frac{1}{2}}(x)\right] . \qquad (206)$$

From Dingle,[34] the series expansion gives (for $x < 1$)

$$\mathfrak{C}_{\frac{3}{2}}(x) = \frac{4}{3}\left\{1 - \frac{4}{3}x^2 + \frac{16}{105}x^4 - \ldots\right\} - \frac{4\sqrt{\pi}\sqrt{x}}{3}\sin(x + \tfrac{3\pi}{4}) \quad (207)$$

$$\mathfrak{C}_{\frac{5}{2}}(x) = \frac{4}{15}\left\{1 + 4x^2 - \frac{16}{15}x^4 + \ldots\right\} + \frac{8\sqrt{\pi}}{15}x\sqrt{x}\sin(x+\tfrac{5\pi}{4}) \quad (208)$$

$$\mathfrak{C}_{\frac{7}{2}}(x) = \frac{4}{35}\left\{1 - \frac{4}{3}x^2 + \frac{16}{9}x^4 - \ldots\right\} - \frac{16\sqrt{\pi}}{105}x^2\sqrt{x}\sin(x+\tfrac{7\pi}{4}) \quad (209)$$

and for $x > 1$,

$$\mathfrak{C}_p(x) = \frac{1}{x^2}\left\{1 - \frac{(p+1)(p+2)}{x^2} + \frac{(p+1)(p+2)(p+3)(p+4)}{x^4} - \ldots\right\}. \quad (210)$$

From Dingle et al [34] for $b \ll 1$, $x = \frac{3}{2b} \gg 1$,

$$\mathfrak{C}_{\frac{5}{2}}(x) \approx \frac{1}{x^2}\left(1 - \frac{63}{4x^2}\right) \qquad (211)$$

$$\mathfrak{C}_{\frac{7}{2}}(x) \simeq \frac{1}{x^2}\left(1 - \frac{99}{4x^2}\right) \qquad (212)$$

From Eqs. (199), (200), (58), (211) and (212),

$$\alpha_1 \simeq \frac{7}{3} - \frac{28}{3} b^2 \qquad (213)$$

$$\alpha_2 \simeq \sqrt{\frac{35}{9}}\left(1 - \frac{25}{9} b^2\right), \quad b << 1 \qquad (214)$$

and

$$\alpha_3 \simeq \frac{\sqrt{35}}{7}\left(1 + \frac{11}{9} b^2\right), \quad b < 1 \qquad (215)$$

and for b >> 1, $x - \frac{3}{2b} << 1$ (from Eq.(208) and Eq.(209))

$$\alpha_1 \simeq 1 + \frac{3}{2b}\sqrt{\frac{3\pi}{b}}, \qquad \alpha_2 \simeq \alpha_1, \qquad (216)$$

$$\alpha_3 \simeq 1 - \frac{3}{2b^2} \qquad (217)$$

The Case of $n = -1$, $Q = Q_0 v^{-2}$, $\nu = NQ_0 v^{-1}$

$$I_1 = \frac{4c_{-1}^{-1}}{3b^2\sqrt{\pi}} \int_0^\infty \frac{W^2 e^{-W} dW}{W + b^{-2}c_{-1}^{-2}} = \frac{2}{b^2} \mathfrak{A}_2\left(\frac{9\pi}{16b^2}\right) \qquad (218)$$

$$I_2 = \frac{4}{3b^2\sqrt{\pi}} \int_0^\infty \frac{W^{5/2} e^{-W} dW}{W + b^{-2}c_{-1}^{-2}} = \frac{5}{2b^2} \mathfrak{A}_{2.5}\left(\frac{9\pi}{16b^2}\right) \quad , \qquad (219)$$

$$\bar{\nu}_{e,n} = \frac{4}{3\sqrt{\pi}} NQ_0 \left(\frac{m_e}{2kT_e}\right)^{1/2} \qquad (220)$$

Dingle et al.[34] related the $\mathfrak{A}_p$ function to the exponential integral

$$\mathfrak{A}_p(x) = e^x E_{p+1}(x) \quad , \qquad (221)$$

where

$$E_{p+1}(x) = \int_1^\infty \frac{e^{-xt}}{t^{p+1}} dt \quad . \qquad (222)$$

From Bierens de Haan[36] and Batman[37] for $x < 1$ if p is

$$\mathfrak{A}_p(x) = \frac{1}{p} \left\{1 - \frac{x}{p-1} + \frac{x^2}{(p-1)(p-2)} - + \cdots + \frac{(-x)^{p-1}}{(p-1)!}\right\} + \frac{(-x)^p}{p!} e^x \left[-\mathrm{Ei}(-x)\right] \quad , \qquad (223)$$

where

$$-\mathrm{Ei}(-x) = \mathrm{ei}(x) = \int_x^\infty \frac{e^{-t}}{t} dt = \int_1^\infty \frac{e^{-xt}}{t} dt \equiv E_1(x) \; ,$$

$$E_1(x) = -\ln\gamma x + x - \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} - \frac{x^4}{4 \cdot 4!} + - \cdots \; ,$$

[illegible] is the Euler's constant. If p is not an inte- [illegible]

$$\mathfrak{A}_p(x) = \frac{1}{p}\left[1 - \frac{x}{p-1} + \frac{x^2}{(p-1)(p-2)} - + \dots\right] - \frac{\pi x^p e^x}{p!\,\sin\pi p} \,. \qquad (224)$$

for $x >> 1$, Dingle quotes[34]

$$\mathfrak{A}_{p-1}(x) = \frac{1}{p+x}\left\{1 + \frac{p}{(p+x)^2} + \frac{p(p-2x)}{(p+x)^4} + \frac{p(p^2-8px+6x^2)}{(p+x)^6} + \frac{p(p^3-22p^2x+58px^2-24x^3)}{(p+x)^8} + \dots\right\} . \qquad (225)$$

Therefore, for $b << 1$, $x = \frac{9\pi}{16b^2} >> 1$,

$$\mathfrak{A}_2(x) \simeq \frac{1}{3+x}\left(1 + \frac{3}{(3+x)^2}\right) \qquad (226)$$

$$\mathfrak{A}_{5/2}(x) \simeq \frac{1}{\frac{7}{2}+x}\left(1 + \frac{7/2}{\left(\frac{7}{2}+x\right)^2}\right) \qquad (227)$$

and

$$\sigma_1 \simeq \frac{5}{4} - \frac{10b^2}{3\pi} \qquad (228)$$

$$\sigma_2 \simeq \sqrt{\frac{40}{9\pi}}\left[1 - \left(\frac{28}{9\pi} - \frac{25}{32}\right)b^2\right] \,. \qquad (229)$$

For [illegible] and $\frac{9\pi}{16b^2} << 1$,

$$\mathfrak{A}_2(x) \simeq \frac{1}{2}(1-x) \qquad (230)$$

$$\mathfrak{A}_{5/2}(x) \simeq \frac{2}{5}\left(1 - \frac{2}{3}x\right) - \frac{8\sqrt{\pi}}{15}\, x^{5/2}\, e^x \qquad (231)$$

$$\sigma_1 \simeq 1 + \frac{3\pi}{16b^2} \qquad (232)$$

$$\sigma_2 \simeq 1 + \dots \qquad (233)$$

The Case of $n = 0$, $Q = Q_0 v^{-1}$, $\nu = NQ_0$ = constant.

$$I_1 = \frac{1}{1+b^2} \qquad , \qquad I_2 = \frac{1}{1+b^2} \tag{234}$$

$$\alpha_1 \quad \alpha_2 = \alpha_3 = 1 \qquad , \qquad \bar{\nu} - NQ_0 \tag{235}$$

$$\sigma' - \frac{\omega_p^2}{\bar{\nu} - i\omega} \tag{236}$$

which is the ordinary conductivity.

The Case of $n = 1$, $Q = Q_0$ = constant, $\nu = NQ_0 v$.

This is a continuation of the case discussed in Eqs. (172) through (175).

$$I_1 = \frac{4}{3\sqrt{\pi}} c_1 \int_0^\infty \frac{W^2 e^{-W} dW}{W + b^2 c_1^2} = \frac{64}{9\pi} \mathfrak{A}_2\left(\frac{64}{9\pi} b^2\right) \tag{237}$$

$$I_2 = \frac{4}{3\sqrt{\pi}} c_1^2 \int_0^\infty \frac{W^{3/2} e^{-W} dW}{W + b^2 c_1^2} = \frac{64}{9\pi} \mathfrak{A}_{3/2}\left(\frac{64}{9\pi} b^2\right) \tag{238}$$

$$\bar{\nu}_{e,n} = \frac{8}{3\sqrt{\pi}} NQ_0 \cdot \left(\frac{2kT_e}{m_e}\right)^{1/2} \tag{239}$$

The function $\mathfrak{A}_p$ was described previously in the $n = -1$ case. For $b \ll 1$, $x = \frac{64}{9\pi} b^2 \ll 1$, $\mathfrak{A}_2(x) \approx \frac{1}{2}(1-x)$;

$$\mathfrak{A}_{3/2}(x) - \frac{2}{3}(1 - 2x) , \tag{240}$$

$$\alpha_1 \simeq \frac{4}{3} - \frac{256}{27\pi} b^2 \tag{241}$$

$$\alpha_2 \simeq \sqrt{\frac{128}{27\pi}} \left[1 + \frac{8}{9}\left(1 - \frac{8}{\pi}\right) b^2\right] \tag{242}$$

TABLE 2

CORRECTIONS TO THE ELECTRIC CONDUCTIVITY FOR RIGID SPHERICAL ATOMS AND MOLECULES IN A WEAKLY IONIZED PLASMA*

(Q – constant)

$\frac{\omega}{\bar{\nu}}$	$\alpha_1 = \frac{(\omega/\nu)_{eff}}{\omega/\bar{\nu}}$	$\alpha_2 = \frac{(\omega_p/\nu)_{eff}}{\omega_p/\bar{\nu}}$	$\alpha_3 = \frac{(\omega_p/\omega)_{eff}}{\omega_p/\omega}$
0	$\frac{4}{3} = 1.333$	$\frac{128}{27\pi} = 1.2284$	$\sqrt{\frac{8}{3\pi}} = 0.9213$
0.05	1.33	1.22	0.923
0.1	1.31	1.22	0.924
0.2	1.28	1.20	0.934
0.5	1.19	1.14	0.956
1.0	1.10	1.08	0.976
2.0	1.04	1.03	0.990
5.0	1.01	1.01	0.998
10.0	1.002	1.002	0.9995
[illegible]	1.000	1.000	0.9999
–	1.000	1.000	1.000

[illegible] $N(\pi r_o^2)$, $\nu_{e,n} \gg \nu_{e,i}$, $Q_{e,n} = \pi r_o^2 = \text{const}$

---

* For more details, see Table A-1 for n = 1

[illegible] $1 - \frac{64}{9\pi} b^2 >> 1$,

$$\mathfrak{A}_2(x) = \frac{1}{x+3} \quad , \qquad \text{[illegible]}$$

$$\mathfrak{A}_{3/2}(x) = \frac{1}{x+\frac{5}{2}} \qquad \text{[illegible]}$$

$$\alpha_1 = 1 + \frac{9\pi}{128b^2} \qquad (245)$$

$$\alpha_2 = \alpha_1 \qquad (246)$$

This case was investigated by Fang[38a] who first pointed out the relation to Dingle integrals $\mathfrak{A}_2$ and $\mathfrak{A}_{1.5}$.

The Case of [illegible], $Q = Q_0 v$, $\nu = NQ_0 v^2$.

$$I_1 = \frac{4}{3\sqrt{\pi}} c_2 \int_0^\infty \frac{W^{5/2} e^{-W} dW}{W^2 + c_2^2 b^2} = \frac{25}{4} \mathfrak{C}_{5/2}\left(\frac{5}{2} b\right) \tag{247}$$

$$I_2 = \frac{4}{3\sqrt{\pi}} c_2^2 \int_0^\infty \frac{W^{3/2} e^{-W} dW}{W^2 + c_2^2 b^2} = \frac{25}{4} \mathfrak{C}_{3/2}\left(\frac{5}{2} b\right) \tag{248}$$

$$\bar{\nu}_{e,n} = 5NQ_0 \cdot \frac{kT_e}{m_e} \quad . \tag{249}$$

The $\mathfrak{C}_p$ function was discussed in n = -2 case.

For $b << 1$, $x - \frac{5}{2}b << 1$,

$$\mathfrak{C}_{3/2}(x) \cong \frac{4}{3}\left[1 - \frac{4}{3}x^2 + \sqrt{\pi}\sqrt{x}\cos\left(x + \frac{\pi}{4}\right)\right] \tag{250}$$

$$\mathfrak{C}_{5/2}(x) \cong \frac{4}{15}\left[1 + 4x^2 - 2\sqrt{\pi}\, x\sqrt{x}\sin\left(x - \frac{\pi}{4}\right)\right] \tag{251}$$

and

$$\alpha_1 \cong 5\left[1 - 25b^2 - \frac{5}{2}\sqrt{\pi}\, b \sin\left(\frac{5}{2}b + \frac{3\pi}{4}\right)\right] \tag{252}$$

$$\alpha_2 \cong \frac{5}{\sqrt{3}}\left[1 + 25b^2 - \frac{5}{4}\sqrt{\pi}\, b \sin\left(\frac{5}{2}b + \frac{3\pi}{4}\right)\right] \tag{253}$$

For $b >> 1$, $x - \frac{5}{2} b >> 1$,

$$\mathfrak{C}_{3/2}(x) \cong \frac{1}{x^2}\left(1 - \frac{35}{4x^2}\right) \tag{254}$$

$$\mathfrak{C}_{\,/2}(x) \cong \frac{1}{x^2}\left(1 - \frac{63}{4x^2}\right) \tag{255}$$

(Note an error in Molmud^{35} in Eq. A-7, 7 should read 63).

$$\alpha_1 \cong 1 + \frac{28}{25b^2} \quad , \qquad \alpha_2 \cong 1 + \frac{23}{25b^2} \tag{256}$$

and

$$\alpha_3 \cong 1 - \frac{1}{5b^2} \tag{257}$$

The Case of $n = 3$, $Q = Q_0 v^2$, $\nu = NQ_0 v^3$

$$I_1 = \frac{4}{3b^2\sqrt{\pi}}\, c_3^{-1} \int_0^{\infty} \frac{W^3 e^{-W} dW}{1 + W^3 c_3^{-2} b^{-2}} = \frac{1}{b^2}\, \mathfrak{F}_3\left(\frac{\pi}{64b^2}\right) \qquad (258)$$

$$I_2 = \frac{4}{3b\sqrt{\pi}} \int_0^{\infty} \frac{W^{3/2} e^{-W} dW}{1 + W^3 c_3^{-2} b^{-2}} = \frac{1}{b^2}\, \mathfrak{F}_{1.5}\left(\frac{\pi}{64b^2}\right) \qquad (259)$$

$$\bar{\nu}_{e,n} = \frac{8}{\sqrt{\pi}} NQ_0 \cdot \left(\frac{2kT_e}{m_e}\right)^{3/2} . \qquad (260)$$

There are no convenient asymptotic series[34] for $\mathfrak{F}_p(x)$ function, and series quoted by Dingle[34] are very slowly convergent (see discussion after Eq. (158)).

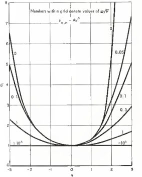

FIGURE 3 Correction α_1 for Weakly Ionized Plasma

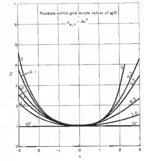

FIGURE 4 Correction α_2 for Weakly Ionized Plasma

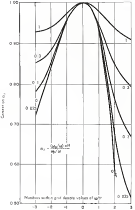

FIGURE 5 Correction α_3 for Weakly Ionized Plasma

The DC and Related Limiting Cases of $\omega \pm \omega_H^* = 0$

For $b = \frac{\omega}{\nu} = 0$, (DC case), (or $b = \frac{\omega \pm \omega_H^*}{\nu} = 0$, see footnote to Table A-1) we have in the limit of $b \to 0$

$$I_1 = \frac{4}{3\sqrt{\pi}} c_n \int_0^\infty W^{\frac{3-n}{2}} e^{-W} dW = \frac{4}{3\sqrt{\pi}}\ c_n\, \Gamma\left(\frac{5-n}{2}\right)$$

$$I_2 = \frac{4}{3\sqrt{\pi}} c_n^{\ 2} \int_0^\infty W^{\frac{3-2n}{2}} e^{-W} dW = \frac{4}{3\sqrt{\pi}}\ c_n^{\ 2}\, \Gamma\left(\frac{5-2n}{2}\right). \tag{261}$$

From Eq. (58) we obtain in the limit of $b \to 0$

$$\alpha_1^o = \frac{I_2}{I_1} = \frac{4}{3\sqrt{\pi}} \frac{\Gamma\left(\frac{5-2n}{2}\right)\Gamma\left(\frac{5+n}{2}\right)}{\Gamma\left(\frac{5-n}{2}\right)}\quad , \quad \alpha_2^o \alpha_3^o = I_1 \tag{262}$$

$$\alpha_2^o \cong \sqrt{I_2} = \frac{4}{3\sqrt{\pi}}\ \Gamma\left(\frac{5+n}{2}\right)\sqrt{\frac{4}{3\sqrt{\pi}}\ \Gamma\left(\frac{5-2n}{2}\right)}. \tag{263}$$

The direct current (d.c.) electric conductivity is given by (see Eq. (60))

$$\sigma_{DC} = \sigma_r = \frac{\omega_p^{\ 2}}{\nu_{DC}} = \frac{\alpha_2^o \alpha_3^o \omega_p^{\ 2}}{\bar{\nu}} = \Gamma\left(\frac{5+n}{2}\right)\Gamma\left(\frac{5-n}{2}\right)\frac{\omega_p^{\ 2}}{\bar{\nu}}, \tag{264}$$

where $\bar{\nu}$ is the average collision frequency at $\omega >> \nu$ and ν_{DC} is the average effective collision frequency at $\omega = 0$. Thus (see Eq. (76))

$$\frac{\nu_{DC}}{\bar{\nu}} = \frac{9\pi}{16} \frac{1}{\Gamma\left(\frac{5+n}{2}\right)\Gamma\left(\frac{5-n}{2}\right)} = \frac{\alpha_1^o}{\alpha_2^{o2}} = \frac{1}{\alpha_2^o \alpha_3^o} = g_\sigma^o = \frac{3\pi}{32\gamma_E} \tag{265}$$

where g_σ is the notation used by Shkarofsky[25] and γ_E by Spitzer[26] (see Eqs. 69 and 76) for fully ionized plasma.

In Table A-1, F-1, F-2 and in Figure 6 are given* some values of $v_{DC}/\bar{\nu}$, and also are given α_1 and α_2, which in the case of $\omega = \omega_H{}^* = 0$ have only importance as an asymptotic values for determination of α's for $|\omega \pm \omega_H{}^*|/\bar{\nu}$ near zero. As we see from Table A-1,

$$\lim_{\omega\to 0} \frac{(\omega/\nu)_{\text{eff}}}{\omega/\bar{\nu}} = \lim_{\omega\to 0} \alpha_1 \neq \frac{\bar{\nu}}{\nu_{DC}} = \frac{\alpha_2{}^2}{\alpha_1} = \alpha_2\alpha_3 \quad . \tag{266}$$

For highly ionized plasma (of low z) at b = 0, the electron-electron collisions are important and one cannot use Margenau theory of weakly ionized plasma by only using electron-ion collision frequency ν_{ei} (see Ch. II, Ref. 60).

From Eq. (51) we have for a weakly ionized plasma

$$\frac{1}{\nu_{DC}} = \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{W^{3/2}\, e^{-W}\, dW}{\nu} \quad , \tag{267}$$

where

$$\nu = \sum_j \nu_{e,n}^{(j)} + \nu_{e,i}$$

If $\nu_{ei} \gtrsim \nu_{e,n}$, the electron-electron collisions are important and Eq. (34), etc., must be abandoned.

---

* For b = 0, for n = 3, $\alpha_1 = \infty$, $\alpha_2 = \infty$, however, in general $\alpha_2\alpha_3 = \frac{\alpha_2}{\alpha_1} = \frac{I_2(1+\alpha_1{}^2b^2)}{I_2/I_1} = I_1(1+\alpha_1{}^2b^2) \xrightarrow{b\to 0} \frac{16}{9\pi}\,\Gamma\left(\frac{5-n}{2}\right)\Gamma\left(\frac{5+n}{2}\right)$, i.e., for n = 3, $\alpha_2\alpha_3 = \frac{32}{3\pi}$, $\frac{\nu_{ec}}{\nu} = \frac{1}{\alpha_2\alpha_3} = \frac{3\pi}{32}$. Similarily for other n, including n = 2-5 for which $\alpha_1 = \alpha_2 = \infty$.

TABLE 3

LIMITING VALUES OF a's FOR WEAKLY IONIZED GASES (DC and Related Cases)*

$[illegible] = Q_0 v^{n-1} \quad , \quad \nu_{c,n} = NQ_0 v^n$

[illegible]	a_1	a_2	a_3	$\nu_{DC}/\bar{\nu}$
[illegible]	$\frac{105}{16} = 6.5625$	$\sqrt{\frac{70}{\pi}} = 4.7203$	$\frac{16}{105}\sqrt{\frac{70}{\pi}} = 0.7193$	$\frac{3\pi}{32} = 0.2945$
[illegible]	3.6398	2.8972	0.7831	0.4408
[illegible]	$\frac{7}{3}$ 2.3333	$\sqrt{\frac{35}{9}}$ 1.9720	$\frac{1}{7}\sqrt{35} = 0.8452$	$\frac{3}{5} = 0.6000$
[illegible]	1.6274	1.4688	0.9026	0.7543
[illegible]	$\frac{5}{4}$ 1.250	$\sqrt{\frac{40}{9\pi}} = 1.1894$	$\frac{4}{5}\sqrt{\frac{40}{9\pi}} = 0.9515$	$\frac{9\pi}{32} = 0.8836$
[illegible]	1.0598	1.0454	0.9864	0.9697
[illegible]	1	1	1	1
[illegible]	1.0679	1.0484	0.9827	0.9697
[illegible]	$\frac{4}{3}$ 1.3333	$\sqrt{\frac{128}{27\pi}} = 1.2284$	$\frac{3}{4}\sqrt{\frac{128}{27\pi}} = 0.9213$	$\frac{9\pi}{32} = 0.8836$
[illegible]	2.0863	1.6631	0.7971	0.7543
[illegible]	5	$\frac{5}{\sqrt{3}} = 2.8867$	$\frac{\sqrt{3}}{3} = 0.5774$	$\frac{3}{5} = 0.6000$
[illegible]		-	-	0.4408
[illegible]		--	--	$\frac{3\pi}{32}$ - 0.2945

---

* This applies to the following cases: (see Eqs. (145) - (148))

1. [illegible] = 0, $a_1 = \alpha_1 = \alpha_1^+ - \alpha_1^-$, a_2 [illegible] α_2, ν_{DC} [illegible] $\frac{\bar{\nu}}{\alpha_1 \alpha_3^2} = \frac{[illegible]}{\alpha_2 \alpha_3}$
(DC Case) a_3 [illegible] α_3 [illegible] α_3^+ [illegible] α_3^-,
σ' [illegible] $\sigma'_{yx} = \sigma'_{yy} = \sigma'_{xx}$ [illegible] ω_p^2/ν_{DC}.

2. [illegible] 0, [illegible] 0 (DC case)(or [illegible] >> [illegible]) $a_1 = \alpha_1^+$ [illegible] α_1^-, $a_3 = \alpha_3^+ = \alpha_3^-$

[illegible] a_1 [illegible] α_1^-, a_3 [illegible] α_3^-

[illegible] 0, [illegible] = 0 (or [illegible] >> ω_c), a_1 [illegible] $\alpha_1 - \alpha_1^+$ [illegible] α_1,
[illegible] α_3 [illegible] α_3^-.

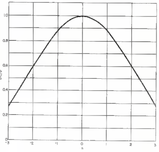

FIGURE 6 The Ratio of Effective (DC) Collision Frequency at $\omega = 0$ to an Average Collision Frequency for $\omega >> \nu$ as a Function of n, where $\nu_{e,n} = A v^n$ for Weakly Ionized Plasma

16 NUMERICAL EVALUATION OF INTEGRALS FOR WEAKLY IONIZED PLASMAS

In the previous section we indicated analytic method for evaluation of integrals for weakly ionized plasma (Eqs. (46) (47) and (61)) in which electron-neutral elastic cross-section for momentum transfer varies as some integral power of velocity. Molmud[44] has also pointed out that the evaluation of the integrals for collision cross-section approximated by finite series expansion in velocity can be reduced to the tabulated error functions and exponential integrals, both of complex arguments.

However, the resulting integrals, even for multicomponent weakly ionized plasma, can be very simply and accurately evaluated numerically by the Simpson formula, or using n point Laguerre[39] formula based on the roots of the Laguerre polynomials, i.e.,

$$\int_0^\infty f(W)e^{-W}dW \sim \sum_{k=1}^{n} A_k\, f(W_k) \quad , \qquad (268)$$

where A_k's and W_k's are precomputed by Salzer and Zucker[39] and are listed in Table 4 for a n=5 point formula.* Table 4 illustrates the use of the formula. For the prescribed value of W_k and a given value of T_e, one computes for j species in a plasma, at first

---

*The n point formula is exact for polynomials up to 2n + 1 degree.

$$W_k(\text{eV}) = \frac{T_e(^\circ K)}{11605} W_k \qquad Q(W_k) = \sum X_j Q_j(W_k) \tag{269}$$

and

$$\nu_k = NQ(W_k)\sqrt{\frac{2kT_e}{m_e}}\sqrt{W_k} + \nu_{ei}(W_k) \quad , \tag{270}$$

where ν_{ei} is the electron ion collision frequency (see Ch. II, Ref. 60) and where Q_j is the momentum transfer elastic cross-section of an electron with jth neutral species and $X_j = N_j/\sum N_j$. Then, for a given value of $\Omega/\bar{\nu}$, where $\Omega = |\omega \pm \omega_c^*|$, from Eqs. (46), (47) and Eq. (49),

$$I_1 = \frac{4}{3\sqrt{\pi}} \sum_{k=1}^{n} A_k \frac{(\nu_k/\bar{\nu}) W_k\sqrt{W_k}}{(\nu_k/\bar{\nu})^2 + b^2} \tag{271}$$

$$I_2 = \frac{4}{3\sqrt{\pi}} \sum_{k=1}^{n} A_k \frac{W_k\sqrt{W_k}}{(\nu_k/\bar{\nu})^2 + b^2} \tag{272}$$

$$\bar{\nu} = \frac{4}{\sqrt{\pi}} \sum_{k=1}^{n} A_k \nu_k W_k \sqrt{W_k} \tag{273}$$

$$a_1 = I_2/I_1 \tag{274}$$

$$a_2 = \sqrt{I_2(1 + a_1^2 b^2)} \tag{275}$$

$$a_3 = a_2/a_1 \tag{276}$$

[illegible] 3)

$$\frac{1}{\nu_k} = \frac{4}{3\sqrt{\pi}} \sum_{k=1}^{n} A_k \frac{W_k\sqrt{W_k}}{\nu_k} \tag{277}$$

Finally,

for $\omega_H^* = 0$, $a_1 = \alpha_1$, $a_2 = \alpha_2$, $a_3 = \alpha_3$, $b = \frac{\omega}{\nu}$, $\alpha_3 = \alpha_2/\alpha_1$,

for $\Omega_+ = \omega + \omega_H^*$, $b = \frac{\Omega_+}{\nu}$, $a_1 = \alpha_1^+$, $a_3 = \alpha_3^+$, $a_2 = \alpha_2^+ = \alpha_2$,

for $\Omega_1 = |\omega - \omega_H^*|$, $b = \frac{\Omega_-}{\nu}$, $a_1 = \alpha_1^-$, $a_3 = \alpha_3^-$, $a_2 = \alpha_2^- = \alpha_2$.

TABLE [illegible]

EXAMPLES [illegible] S-LAGUERRE QUADRATURE FORMULA

$$I \int_0^\infty f(W)e^{-W}dW \approx \sum_{k=1}^{5} A_k f(W_k)$$

5 point formula:

k	W_k	A_k
1	0.263560	0.521756
2	1.413403	0.398667
3	3.596426	$7.594 4 \times 10^{-[illegible]}$
4	7.085810	$3.61176 \times 10^{-[illegible]}$
5	12.640801	$2.33700 \times 10^{-[illegible]}$

Examples (in using above truncated values of W_k and A_k)

$f(W)$	exact integral	5 point formula	1[illegible]
W	1	0.[illegible]	1.00[illegible]
W^2	2	[illegible]	[illegible]
W^3	6	[illegible]9991	6.[illegible]
W^4	24	24.0000	[illegible]
W^5	120	119.998	1[illegible]
W^6	720	71[illegible]	[illegible]
$W^{3/2}$	1.[illegible]	1.3[illegible]76	1.[illegible]
$W^{5/2}$	3.[illegible]	[illegible]	[illegible]
$W^{7/2}$	11.[illegible]	1[illegible]	[illegible]
$W^{9/2}$	[illegible]	[illegible]	[illegible]
$W^{11/2}$	287.88[illegible]	[illegible]	[illegible]
$W^{-1/2}$	1.77[illegible]	1.[illegible]	[illegible]
$(1+W)^{-1}$	0.[illegible]	0.[illegible]	[illegible]
$W(1-e^{-W})^{1/2}$	0.85557	0.8[illegible]16	[illegible]
$W(1-e^{-2W})^{1/2}$	[illegible]	[illegible]	[illegible]
$\ln W$	0.[illegible]	[illegible]	[illegible]
$\cosh \frac{1}{2}W$	[illegible]	[illegible]	[illegible]
$\sin W$	[illegible]	0.[illegible]	[illegible]

---

* 15 point formula [illegible]

TABLE 5

ENERGIES AT WHICH CROSS SECTIONS ARE NEEDED FOR THE EVALUATION OF CONDUCTIVITY INTEGRALS FOR WEAKLY IONIZED GASES
(MAXWELLIAN VELOCITY DISTRIBUTION)

(FIVE POINT LAGUERRE INTEGRATION)

E_k (in ev)

W_k	$T=300^\circ K$	$T=1000^\circ K$	$T=2000^\circ K$	$T=3000^\circ K$	$T=4000^\circ K$	$T=5000^\circ K$	$T=6000^\circ K$	$T=7000^\circ K$	$T=10^4\ ^\circ K$
0.26356	0.00681	0.0227	0.0454	0.0681	0.0908	0.1135	0.1362	0.1589	0.2270
1.4134	0.0365	0.1218	0.2436	0.3654	0.4873	0.6091	0.7309	0.8527	1.2182
3.5964	0.0929	0.3099	0.6198	0.9297	1.240	1.549	1.859	2.169	3.099
7.0858	0.1831	0.6105	1.2210	1.8315	2.4420	3.0525	3.6630	4.2735	6.105
12.6408	0.327	1.089	2.178	3.267	4.357	5.446	6.535	7.624	10.892

$$E_k \text{(in ev)} = \frac{W_k}{1.16} T \times 10^{-4}$$

(The 10 point formula gives energy range greater by about a factor of two.)

On Uncertainties in the Evaluation of Integrals Involving Electron-Neutral Collision CrossSections for Momentum Transfer as a Function of Electron Energy

The uncertainty in the electron-neutral elastic cross sections for the momentum transfer* is about ± 0.05 x 10^{-15}cm^2 from a given experiment (e.g., molecular beam). The initial collection of results from the various methods of measurements of cross sections by various investigators[45] and from the various methods of calculations give different values (in the range of 0 to 2 ev), e.g., a factor up to 7 for N_2, of 3 for O_2, of 2 for NO, of 3 for N, of 8 for O, of 9 for A, etc. However, after careful review one decides on the most reliable measurement and theoretical calculation. (See Appendix G). Then the remaining uncertainty at best is still ± 0.05 x 10^{-15}cm^2. (Assuming constant with energy $\Delta Q = \pm 0.05 \times 10^{-15}$cm^2 gives* $\Delta\bar{\nu} = \pm 0.2 \times 10^{11} s/p_0$ at $T = 300°K$.) This limits the accuracy of integration at most to 2 or 3 significant digits. The five point Laguerre formula (Salzer and Zucker[38]) is definitely sufficient (see Table 4). Table 5 indicates the values of energy E_{ev}, at which the cross sections are needed for the evaluation of the integrals (assuming 5 point formula is sufficient) for various temperatures.

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* See Table 6, s=o, $Y_s = 3.855 \times 10^{26}$, $\Delta\bar{\nu} = 3.855 \times 10^{26} \cdot 5 \times 10^{-17} s/p_0$

[illegible] CONTRIBUTION OF A RESONANCE I[illegible]
TO COLLISION FREQUENCY

Let the momentum transfer elastic [illegible] electron with neutral atom or molecule be split [illegible] one, without a resonance, which we approximate [illegible] polynomial fit in energy and a resonant part at $E = E_o$, which we approximate by a Gaussian curve, i.e.

$$Q(E) = \sum_s q_s E_{eV}^{\,s} + Q_r e^{-a(E-E_o)^2} \qquad (278)$$

Then, for $\omega \gg \nu$ from Eq. 61 and from Eq. 180,

$$\bar{\nu}_{e,n} = \frac{4}{\sqrt{\pi}} N \sqrt{\frac{2kT_e}{m_e}} \sum_s q_s \left(\frac{T_e}{11605}\right)^s \Gamma(s+3) + \bar{\nu}_{e,n}^{\,res} \,, \qquad (279)$$

where the contribution of resonance cross-section to the collision frequency is

$$\bar{\nu}_{e,n}^{\,res} = \frac{4}{\sqrt{\pi}} N \sqrt{\frac{2kT_e}{m_e}}\, Q_r I \,, \qquad (280)$$

$$I = \int_0^\infty W^2 e^{-W} F(W)\, dW \,, \quad W = \frac{E}{T} \,, \quad T = \frac{T_e}{11605} \,, \qquad (281)$$

where T_e is in $^\circ K$, E in electron volts and where

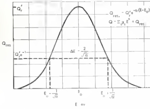

FIGURE 7 Assumed Gaussian Profile for the Resonance in a Cross-Section

$$F(W) = \exp\left[-\alpha t^2 (W - W_0)^2\right] . \tag{282}$$

Given an exact integration* we have

$$e^{-\alpha E_0^2}\,\frac{\alpha t E_0 - \frac{1}{2}}{2\alpha^2 t^4} + \sqrt{\pi}\, e^{\frac{\frac{1}{4} - \alpha t E_0}{\alpha t^2}}\;\frac{\alpha t^2 + 2(\frac{1}{2} - \alpha t E_0)^2}{4\alpha^{5/2} t^5}\,\mathrm{Erf}\left(\frac{\frac{1}{2} - \alpha t E_0}{t\sqrt{\alpha}}\right), \tag{283}$$

where Erf (x) is the error function

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}\,du = 1 - E_2(x) , \tag{284}$$

$$E_2(x) = \frac{2}{\sqrt{\pi}}\int_0^{x} e^{-u^2}\,du \tag{285}$$

If we define the width of the resonance ΔE by (see sketch)

$$Q(E_0 - \tfrac{1}{2}\Delta E) = Q(E_0 + \tfrac{1}{2}\Delta E) = Q_r e^{-1} , \tag{286}$$

then

$$\Delta E = \frac{2}{\sqrt{\alpha}} . \quad \text{Thus for} \quad \Delta E < 1 \text{ ev} , \quad \alpha \geq 4 . \tag{28\ }$$

$$^{*}\int_0^{\infty} x^2 e^{-px^2 - 2ax}\,dx = \frac{a}{p^2} + \sqrt{\pi}\, e^{a^2/p}\,\frac{p + 2a^2}{4p^{5/2}}\,\mathrm{Erf}\left(\frac{a}{\sqrt{p}}\right) . \tag{283a}$$

(This integral can be obtained by a substitution $u = x\sqrt{p} + a/\sqrt{p}$, forming e^{-u^2} and using $\int_\beta^{\infty} u e^{-u^2}\,du = \frac{1}{2} e^{-\beta^2}$ and $\int_\beta^{\infty} u^2 e^{-u^2}\,du = \frac{1}{2}\beta e^{-\beta^2} + \frac{1}{2}\int_\beta^{\infty} e^{-u^2}\,du$, see also J Bateman,[37] pg 146, Eq 3.)

For a very narrow resonance ($\alpha >> 1$) one can show that if $\frac{\Delta E}{E_o} << \sqrt{\alpha}$ and if $\frac{\Delta E}{E_o} << \frac{2T_e}{11605\Delta E}$ then,

$$I \sim \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\,\frac{E_o^{\,2}}{t^3}\,e^{-E_o/t} \tag{288}$$

and

$$\tilde{\nu}_{e,n}^{\,res} \approx \frac{2}{3}\,N\sqrt{\frac{2kT_e}{m_e}}\left(\frac{11605}{T_e}\right)^3 \frac{Q_r E_o^{\,2}}{\sqrt{\alpha}}\,e^{-\frac{11605\,E_o}{T_e}}\ , \ (\alpha >> 1), \tag{289}$$

where T_e is in ^oK and E_o in electron volts. A very narrow resonance in the cross-section contributes maximum to the average collision frequency at the energy equal to $\frac{5}{2}$ kT, i.e., at the temperature

$$T_e(^oK) = 4{,}642.4\ E_o(ev) \quad .$$

Thus, if the narrow ($\alpha >> 1$) resonance is at

$E_o \simeq 0.02 - 0.06$ ev , $\nu_{e,n}^{res}$ has maximum at $T \sim 100^oK - 300^oK$

$E_o \simeq 0.2 - 0.65$, $\nu_{e,n}^{res}$ has maximum at $T \sim 1000 - 3000^oK$

$E_o \gtrsim 2$ ev , $\nu_{e,n}^{res}$ has maximum for $T \gtrsim 10{,}000^oK$.

At such maximum the contribution of the resonance to the collision frequency is

$$(\bar{\nu}_{e,n}^{\,res})_{max} \simeq 3.2 \times 10^7\ Q_r N \frac{\Delta E}{\sqrt{E_o}} \qquad (\Delta E < 1\ ev) \tag{290}$$

where N is in cm^{-3}, Q_r in cm^{-2} and ΔE is defined as in the sketch. For a given temperature, the maximum contribution to $\bar{\nu}_{e,n}$ as a result of a very narrow resonance occurs at energy of 2kT (W = 2), i.e., if $E_o(ev) \simeq \frac{T}{5800}$. If the non-resonant portion of the cross-

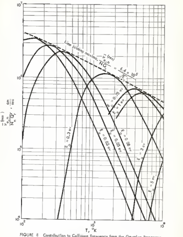

FIGURE 8 Contribution to Collision Frequency from the Gaussian Resonance in a Cross-Section $\alpha = 10$ ($\Delta E = 0.02$ ev), Various E_o

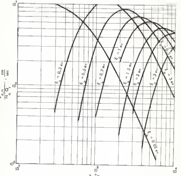

FIGURE 9 Average Electron Collision Frequency from the Gaussian Resonance in a Cross-Section. $\alpha = 10^2$ ($\Delta E = 0.2$ ev)

section is a constant Q_0, i.e., if

$$Q = Q_0 + Q_r e^{-\alpha(E-E_0)^2} \quad , \qquad (291)$$

then the fractional contribution of a narrow resonance to the total collision frequency is

$$\frac{\Delta\bar{\nu}}{\bar{\nu}} = \frac{\lambda}{2+\lambda} \quad , \quad \lambda \approx \frac{\sqrt{\pi}}{2} \left(\frac{11605}{T}\right)^3 \frac{Q_r}{Q_0} \frac{E_0{}^2}{\sqrt{\alpha}} \; e^{-\frac{11605\,E_0}{T_e}} \quad . \qquad (292)$$

In deducing electron neutral elastic cross-section from the experimentally obtained $\bar{\nu}_{e,n}$ (e.g., at $\omega \gg \nu$ from microwave measurements) one usually assumes one or two values of s in Eq.(278) over the range of temperature measured and fits the experimental data of $\bar{\nu}_{e,n}$. As one can see it here, this will only give good results in the absence of a resonance in the cross-section.

If $x = \frac{1}{2t\sqrt{\alpha}} - E_0\sqrt{\alpha} \gg 1$, i.e., if $T_e \ll \frac{11605}{2\sqrt{\alpha}(1+E_0\sqrt{\alpha})}$,

then from expansion of an error function

$$\bar{\nu}_{e,n}^{\,res} = \frac{4}{3\sqrt{\pi}} \sqrt{\frac{2kT_e}{m_e}} \left\{ \frac{1}{4\alpha^2 D} - \left(\frac{D}{2\alpha^2} + \frac{1}{4\alpha^2 D}\right) \left[\frac{1}{2x^2} - \frac{1\cdot 3}{4x^4} + - \right] \right\} \quad , \qquad (293)$$

where

$$D = \frac{1}{2} - \alpha t E_0 = xt\sqrt{\alpha}$$

Numerical Fits to Elastic Neutral Cross Sections and the Contribution of Cross Section Near Zero Energy to the Average Collision Frequency

The range of possible power exponents of $\nu = Av^n$, i.e., $Q_n = Q_0 v^{n-1}$ can be approximately deduced from the following consideration. Since (Eq.(168))

$$Q_n = \frac{4\pi}{k^2} \sum_{\ell} (\ell+1) \sin^2(\eta_\ell - \eta_{\ell+1}) \leq \frac{4\pi}{k^2} A \quad , \quad k = \frac{mv}{\hbar} \tag{294}$$

it follows that with a finite $A = \sum_{\ell=0}^{\ell=\ell_{max}} (\ell+1)$

$$Qv^2 \leq \frac{4\pi\hbar^2}{m_e^2} A \quad , \tag{295}$$

or

$$Q_{cm^2} \cdot E_{ev} \leq 5.2 \times 10^{-15} A \ . \tag{296}$$

At very low energies ($E < 1$ ev) only few partial waves contribute to s, mostly the s wave ($\ell = 0$) and the p wave ($\ell = 1$), so that only approximations (from zero energy to some E_0 at which measurements are available) with $n = -1, 0, 1, 2$, etc., are permissible for all energy range, but not with $n < -1$, i.e., not with $n = -2, -3$, etc. (e.g., for $n = -2$, $Q_n v^2 = Q_0 v^{-1}$ can be greater than any constant at some low v). Even though such fits up to zero velocity are not possible generally, one is justified in using any power fit to experimentally observed cross sections because very low velocities (energies) do not contribute significantly to the value of the needed integrals (e.g., in Eq. (35) and (61) we multiply Q by $v^n \exp(-\frac{1}{2}mv^2/kT)$ and integrate over v.

At higher energies, contributions from higher partial waves become important, so that n = 0, 1, 2, 3, ... may be a possible fit to the actual cross-sections up to high energies, beyond whith the exponential weighting factor in Eqs. (35) and (61) would reduce any difference in the value of the definite integrals between these approximations and the true shapes of the cross-section.

Almost all measured elastic cross sections start from some energy E_1, leaving the uncertainty in the range $0 \le E < E_1$. If we assume that the cross section is infinite* at zero energy and that

$$Q_n = \frac{q_p}{E^p} \ , \quad (n = 1-2p,\ \nu = Av^n) \ , \quad 0 \le E < E_1 \ , \qquad (297)$$

then the contribution of this energy range to the average collision frequency, $\bar{\nu}_{e,n}$ is

$$\Delta\bar{\nu}_{e,n} = \frac{4}{3\sqrt{\pi}} N \sqrt{\frac{2kT_e}{m_e}} \left(\frac{11605}{T}\right)^p q_p \int_0^{W_1} W^{2-p} e^{-W} dW \ , \qquad (298)$$

$$W_1 = \frac{11605 \cdot E_1(ev)}{T(^\circ K)} \ , \quad Q(E_1) = \frac{q_p}{{E_1}^p} \ , \qquad (299)$$

$$\Delta\bar{\nu}_{e,n} = \frac{4}{3\sqrt{\pi}} N \sqrt{\frac{2kT_e}{m_e}} \ Q(E_1) \ {W_1}^p \ \Gamma(3-p, W_1) \ , \qquad (300)$$

where

---

* i.e., that there is a resonance at energy very close to E = 0

$$\Gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt \qquad (301)$$

is the incomplete gamma function This gives for p 1, n = -1),

$$\Delta\bar{\nu}_{a,n}^{(max)} = \frac{4}{3\sqrt{\pi}}\, n \sqrt{\frac{2kT_e}{m_e}}\, Q(E_1) \cdot W_1\left[1-(1+W_1)e^{-W_1}\right] \qquad (302)$$

In view of the previous argument, the cross section with a finite number of partial wave contributions cannot go to zero faster than 1/E, and the above expression, it can be shown, will give for $0 < n \leq 1$ maximum contribution to the collision frequency from the 0 to E_1 energy interval. (It is interesting to note that only for $p \leq 5/2$, i.e., $n \geq -4$, one obtains finite $\Delta\bar{\nu}_{a,n}$.)

18. AVERAGE ELECTRON COLLISION FREQUENCY IN TERRESTIAL ATMOSPHERE
($\omega >> \nu$)

The largest error in a calculation of the electron collision frequency for an upper atmosphere as a function of an altitude is associated with the choice of the pressure for a given altitude and the uncertainty of a composition and electron temperature at high altitudes.

In Table 6 we indicate, as an example, a calculation for a particular model. Figure 10 indicates comparisons with calculations by Budden[55]. The atmosphere composition was taken from Refs. 56, 57 and 58. The electron ion collision frequency is that given by[60]

$$\nu_{e,i}(v) = \frac{4\pi}{v^3}\left(\frac{e^2}{m_e}\right)^2 N_e \ln\left(1 + \frac{m_e v^2}{e^2}\lambda_d\right)$$

where λ_d is the Debye length,

$$\lambda_d = \sqrt{\frac{kT}{8\pi N_e e^2}} \simeq 4.88\sqrt{\frac{T}{N_e}} \text{ , cm} \quad (T_e = T_i) .$$

For $\omega >> \nu$

$$\bar{\nu} = \nu_{e,n} + \nu_{e,i} \quad ,$$

where

$$\bar{\nu}_{e,i} \simeq 3.633 \frac{N_e}{T_e\sqrt{T_e}} \ln\left(1 + \frac{3279\, T_e\sqrt{T_i}}{\sqrt{N_e}}\eta\right) \quad ,$$

TABLE 6

AVERAGE ELECTRON COLLISION FREQUENCY FOR AN ATMOSPHERIC MODEL

$(\omega \gg \nu)$

h, km	T, °K	$\bar{\nu}_{e,n}$, sec^{-1}	$\bar{\nu}_{e,i}$, sec^{-1}	$\bar{\nu}$, sec^{-1} $\bar{\nu} = \bar{\nu}_{e,n} + \bar{\nu}_{e,i}$	
0	288	1.71×10^{11}	-	1.71×10^{11}	
10	217	4.21×10^{10}	--	4.21×10^{10}	
20	217	9.27×10^{9}	--	9.27×10^{9}	
30	232	2.03×10^{9}	--	2.03×10^{9}	
40	241	4.57×10^{8}	--	4.57×10^{8}	
50	283	1.44×10^{8}	--	1.44×10^{8}	
60	256	4.25×10^{7}	--	4.25×10^{7}	
70	212	6.72×10^{6}	--	6.72×10^{6}	
80	197	2.32×10^{6}	--	2.32×10^{6}	
90	197	3.11×10^{5}	--	3.11×10^{5}	3.39×10^{5}
100	300	6.73×10^{4}	--	6.73×10^{4}	6.50×10^{4}
110	390	1.71×10^{4}	--	1.71×10^{4}	1.51×10^{4}
120	480	5.08×10^{3}	-	5.08×10^{3}	8.20×10^{3}
130	570	1.81×10^{3}	--	1.81×10^{3}	2.10×10^{3}
140	660	7.79×10^{2}	--	7.81×10^{2}	11.0×10^{2}
160	840	2.30×10^{2}	--	2.32×10^{2}	
180	1100	7.53×10^{1}	2.2	7.75×10^{1}	
220	1200	2.76×10^{1}	11.4	3.90×10^{1}	
250	1200	1.20×10^{1}	5.82×10^{1}	7.02×10^{1}	
300	1200	3.74	2.10×10^{2}	2.14×10^{2}	
350	1200	1.35	4.29×10^{2}	4.30×10^{2}	
400	1200	5.40×10^{-1}	6.33×10^{2}	6.34×10^{2}	
500	1200	9.92×10^{-2}	3.15×10^{2}	3.15×10^{2}	
550	1200	4.54×10^{-2}	2.21×10^{2}	2.21×10^{2}	
600	1200	2.14×10^{-2}	1.57×10^{2}	1.57×10^{2}	
650	1200	1.02×10^{-2}	1.11×10^{2}	1.11×10^{2}	
700	1200	4.98×10^{-3}	7.82×10^{1}	7.82×10^{1}	
750	1200	2.45×10^{-3}	5.48×10^{1}	5.48×10^{1}	

Based on

$0 \le h \le 90$ km, constant composition, ICAS Standard Atmosphere from Handbook of Chemistry and Physics, 43d ed., pg. 3422 (1961-1962).

$100 \le h \le 750$ km, from L. R. Megill and N. P. Carleton.[58]
Assumption $T_e = T_i = T$

Last column: $\bar{\nu}$ based on atmosphere model by Nicolet,[57] with $T_e = T_g$ and his T_g (see Fig. 10)

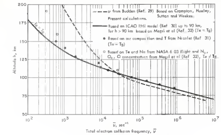

FIGURE 10 The Dependence of the Total Average Electron Collision Frequency $\bar{\nu}$ ($\omega >> \nu$) in Earth Atmosphere as a Function of Altitude

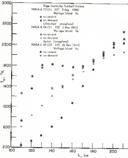

FIGURE 11 Measured Electron Temperature, T_e vs Altitude (from Brace, L.H., et al., J. Geophys. Res. 68, 5397 (1963)).

where T_e and T_i are electron and ion temperatures, respectively, N_e is the electron concentration per cm^3 and where

$$\eta = \sqrt{\frac{2T_e}{T_e + T_i}} \quad .$$

In equilibrium, $T = T_e = T_i$ and $\eta = 1$. The expression for $\bar{\nu}_{e,i}$ holds for $kT_e < e^2/d$, i.e., for $T_e > 5 \times 10^{-3} N_e^{1/3}$

In Fig. 11 is given a particular interpretation of the electron temperature from a dumbbell probe which is still questionable.

APPENDIX A

TABLE A-1

CORRECTIONS TO DIELECTRIC CONSTANTS OF WEAKLY IONIZED GASES FOR VARIOUS VELOCITY DEPENDENCES OF ELECTRON NEUTRAL COLLISION FREQUENCY

(MAXWELLIAN VELOCITY DISTRIBUTION)

$n = -3^*$ $(\nu = Av^n,\ Q = q_s E_{ev}^s = q_n v^{n-1},\ n = 2s+1)$

$\nu_{sc}/\bar{\nu} = 3\pi/32 = 0.2945$

b	a_1	a_2	a_3
0	6.5625	4.7203	.7193
.001	6.5619	4.7201	.7193
.025	6.2945	4.5927	.7296
.050	5.8340	4.3626	.7478
.075	5.4098	4.1398	.7653
.100	5.0528	3.9445	.7807
.200	4.1185	3.3918	.8252
.300	3.5757	3.0915	.8534
.400	3.2261	2.8172	.8732
.500	2.9767	2.6436	.8881
.600	2.7881	2.5985	.8997
.700	2.6395	2.3995	.9091
.800	2.5187	2.3092	.9168
.900	2.4183	2.2329	.9234
1.000	2.3332	2.1674	.9289
1.500	2.0451	1.9389	.9481
2.000	1.8755	1.7992	.9593
5.000	1.4899	1.4651	.9834
10.000	1.3123	1.3023	.9924
25.000	1.1708	1.1680	.9976
50.000	1.1079	1.1068	.9991
100.000	1.0680	1.0676	.9997
1000.000	1.0146	1.0146	1.0000

---

* If $\frac{\nu}{\bar{\nu}} = b$, then $\alpha_1 = a_1$, $\alpha_2 = a_2$, $\alpha_3 = a_3$, $(\alpha_3 = \frac{\alpha_2}{\alpha_1})$.

If $\frac{\omega + \omega_{H_e}}{\bar{\nu}} = b$, then $\alpha_1^+ = a_1$, $\alpha_3^+ = a_3$, $(\alpha_2^+ = \alpha_2 = a_2)$.

If $\left|\frac{\omega - \omega_{H_e}}{\bar{\nu}}\right| = b$, then $\alpha_1^- = a_1$, $\alpha_3^- = a_3$, $(\alpha_2^- = \alpha_2 = a_2)$

TABLE A-1 (Cont)

n = -2.5

	a_1	a_2	a_3
.000	3.8988	2.8972	.7831
.025	3.6832	2.8778	.7858
.050	3.5729	2.8264	.7916
.075	3.4565	2.7871	.7986
.100	3.3418	2.7010	.8083
.200	2.9363	2.4817	.8384
.300	2.6490	2.2916	.8611
.400	2.4428	2.1456	.8783
.500	2.2869	2.0386	.8918
.600	2.1646	1.9542	.9027
.700	2.0464	1.8838	.9118
.800	1.9851	1.8245	.9191
.900	1.9108	1.7739	.9255
1.000	1.8581	1.7269	.9310
1.500	1.6571	1.5745	.9501
2.000	1.5377	1.4784	.9614
5.000	1.2692	1.2506	.9854
10.000	1.1524	1.1453	.9939
25.000	1.0684	1.0847	.9983
50.000	1.0383	1.0357	.9984
100.000	1.0189	1.0187	.9998
1000.000	1.0020	1.0028	1.0000

$v_{00}/\bar{v} = 0.4408$

TABLE A-1 (Cont)

$n = -2.0$

t	a_1	a_2	a_3
.000	2.3333	1.9720	.8452
.025	2.3276	1.9686	.8456
.050	2.3115	1.9590	.8476
.075	2.2868	1.9445	.8503
.100	2.2567	1.9264	.8556
.200	2.1180	1.8409	.8692
.300	1.9896	1.7562	.8840
.400	1.8818	1.6872	.8968
.500	1.7939	1.6274	.9072
.600	1.7213	1.5768	.9161
.700	1.6606	1.5338	.9258
.800	1.6092	1.4966	.9300
.900	1.5651	1.4643	.9356
1.000	1.5269	1.4360	.9405
1.500	1.3929	1.3368	.9576
2.000	1.3122	1.2666	.9678
5.000	1.1344	1.1219	.9890
10.000	1.0645	1.0599	.9959
25.000	1.0217	1.0208	.9991
50.000	1.0080	1.0087	.9997
100.000	1.0065	1.0035	.9996
1000.000	1.0001	1.0001	1.0000

$$\nu_{ec}/\bar{\nu} = m_1/m_2^2 = \frac{3}{5} - 0.6000$$

TABLE A-1 (Cont)

$n = -1.5$

[illegible]	a_1	a_2	a_3
.500	1.6274	1.4688	.9026
.025	1.6281	1.4681	.9028
.050	1.6260	1.4661	.9033
.075	1.6179	1.4627	.9041
.100	1.6111	1.4583	.9052
.200	1.5722	1.4327	.9113
.300	1.5257	1.4014	.9188
.400	1.4798	1.3701	.9259
.500	1.4379	1.3409	.9325
.600	1.4009	1.3145	.9385
.700	1.3676	1.2910	.9438
.800	1.3390	1.2700	.9485
.900	1.3158	1.2513	.9528
1.000	1.2911	1.2347	.9583
1.500	1.2101	1.1782	.9695
2.000	1.1605	1.1544	.9775
5.000	1.0582	1.0491	.9936
10.000	1.0214	1.0192	.9978
25.000	1.0050	1.0148	.9999
50.000	1.0015	1.0014	.9999
100.000	1.0004	1.0094	1.0000
1000.000	1.0000	1.0090	1.0009

$\psi_{oc}/\bar{\psi} = a_1/a_2^2 = 0.7543$

TABLE A-1 (Cont.)

$n = -1.$

	a_1	a_2	a_3
.000	1.2500	1.1884	.8515
.025	1.2496	1.1893	.8516
.050	1.2491	1.1888	.8517
.075	1.2480	1.1880	.8519
.100	1.2465	1.1870	.8522
.200	1.2371	1.1803	.8541
.300	1.2258	1.1708	.8567
.400	1.2086	1.1600	.8598
.500	1.1881	1.1488	.8626
.600	1.1701	1.1378	.8656
.700	1.1640	1.1275	.8686
.800	1.1511	1.1178	.8712
.900	1.1303	1.1082	.8735
1.000	1.1286	1.1012	.8757
1.500	1.0889	1.0710	.8835
2.000	1.0645	1.0520	.8883
5.000	1.0174	1.0144	.9671
10.000	1.0052	1.0044	.9892
25.000	1.0008	1.0008	.9988
50.000	1.0002	1.0002	1.0000
100.000	1.0001	1.0000	1.0000
1000.000	1.0000	1.0000	1.0000

$$v_{bc}/\bar{v} = \frac{9\pi}{32} = 0.8836$$

TABLE A-1 (Cont)

$n = -0.5$

D	a_1	a_2	a_3
.000	1.0598	1.0454	.9884
.025	1.0598	1.0454	.9884
.050	1.0597	1.0453	.9884
.075	1.0595	1.0452	.9885
.100	1.0593	1.0450	.9885
.200	1.0576	1.0437	.9880
.300	1.0550	1.0418	.9875
.400	1.0519	1.0395	.9882
.500	1.0484	1.0369	.9888
.600	1.0446	1.0342	.9888
.700	1.0413	1.0318	.9897
.800	1.0379	1.0290	.9914
.900	1.0346	1.0268	.9922
1.000	1.0318	1.0244	.9928
1.500	1.0208	1.0160	.9953
2.000	1.0142	1.0110	.9968
5.000	1.0030	1.0023	.9993
10.000	1.0006	1.0006	.9998
25.000	1.0001	1.0001	1.0000
50.000	1.0000	1.0000	1.0000
100.000	1.0000	1.0000	1.0000
1000.000	1.0000	1.0000	1.0000

$v_{sc}/\bar{v} = a_1/a_2^2 = 0.9697$

TABLE A-1 (Cont.)

$n = 0$

D	a_1	a_2	a_3
.000	1.0000	1.0000	1.0000
.025	1.0000	1.0000	1.0000
.050	1.0000	1.0000	1.0000
.075	1.0000	1.0000	1.0000
.100	1.0000	1.0000	1.0000
.200	1.0000	1.0000	1.0000
.300	1.0000	1.0000	1.0000
.400	1.0000	1.0000	1.0000
.500	1.0000	1.0000	1.0000
.600	1.0000	1.0000	1.0000
.700	1.0000	1.0000	1.0000
.800	1.0000	1.0000	1.0000
.900	1.0000	1.0000	1.0000
1.000	1.0000	1.0000	1.0000
1.500	1.0000	1.0000	1.0000
2.000	1.0000	1.0000	1.0000
5.000	1.0000	1.0000	1.0000
10.000	1.0000	1.0000	1.0000
25.000	1.0000	1.0000	1.0000
50.000	1.0000	1.0000	1.0000
100.000	1.0000	1.0000	1.0000
1000.000	1.0000	1.0000	1.0000

$v_{sc}/\bar{v} = 1$

TABLE A-1 (Cont)

n = 0.5

b	a_1	a_2	a_3
.000	1.0679	1.0494	.9827
.025	1.0678	1.0493	.9827
.050	1.0676	1.0492	.9828
.075	1.0672	1.0489	.9829
.100	1.0666	1.0488	.9830
.200	1.0636	1.0464	.9838
.300	1.0592	1.0433	.9849
.400	1.0542	1.0397	.9862
.500	1.0491	1.0360	.9876
.600	1.0441	1.0324	.9888
.700	1.0395	1.0291	.9900
.800	1.0354	1.0261	.9910
.900	1.0317	1.0234	.9920
1.000	1.0284	1.0210	.9928
1.500	1.0171	1.0127	.9957
2.000	1.0110	1.0082	.9972
5.000	1.0021	1.0018	.9995
10.000	1.0006	1.0004	.9999
25.000	1.0001	1.0001	1.0000
50.000	1.0000	1.0000	1.0000
100.000	1.0000	1.0000	1.0000
1000.000	1.0000	1.0000	1.0000

$u_{*c}/\bar{v} = 0.3637$

TABLE A-1 (Cont)

$n = 1.0$

b	a_1	a_2	a_3
.000	1.3333	1.2284	.8213
.025	1.3317	1.2275	.8218
.050	1.3274	1.2250	.8228
.075	1.3214	1.2214	.9244
.100	1.3142	1.2171	.8261
.200	1.2803	1.1962	.8342
.300	1.2462	1.1742	.8423
.400	1.2155	1.1540	.8484
.500	1.1888	1.1360	.8558
.600	1.1660	1.1205	.8610
.700	1.1468	1.1070	.8654
.800	1.1300	1.0953	.8683
.900	1.1150	1.0853	.8728
1.000	1.1058	1.0766	.8755
1.500	1.0620	1.0470	.8851
2.000	1.0416	1.0311	.8902
5.000	1.0086	1.0063	.8880
10.000	1.0022	1.0017	.8885
25.000	1.0004	1.0003	.8888
50.000	1.0001	1.0001	1.0000
100.000	1.0000	1.0000	1.0000
1000.000	1.0000	1.0000	1.0000

$v_{bc}/\bar{v} = \frac{9\pi}{32} = 0.8836$

TABLE F-1 (Cont)

1.

b	a_1	a_2	a_3
.000	2.0866	1.6681	.7971
.025	2.0247	1.6381	.8081
.050	1.9580	1.6045	.8195
.075	1.8969	1.5742	.8299
.100	1.8419	1.5458	.8392
.200	1.6708	1.4516	.8808
.300	1.5521	1.3814	.8980
.400	1.4649	1.3273	.9061
.500	1.3980	1.2844	.9187
.600	1.3453	1.2497	.9289
.700	1.3027	1.2210	.9373
.800	1.2676	1.1970	.9443
.900	1.2383	1.1765	.9502
1.000	1.2139	1.1592	.9552
1.500	1.1529	1.1009	.9720
2.000	1.0890	1.0589	.9810
5.000	1.0199	1.0153	.9959
10.000	1.0052	1.0041	.9989
25.000	1.0008	1.0008	.9998
50.000	1.0001	1.0001	1.0000
100.000	1.0000	.9999	1.0000
1000.000	.9999	.9999	1.0000

$v_{sc}/V = 0.7543$

TABLE A-1 (Cont)

[illegible]

[illegible]	a_1	a_2	a_3
.000	4.0097	2.8000	.5774
.025	3.6016	2.4279	.6741
.050	3.1866	2.2541	.7113
.075	2.8914	2.1327	.7376
.100	2.6804	2.0387	.7581
.200	2.2889	1.7942	.8124
.300	1.9477	1.6475	.8456
.400	1.7781	1.5400	.8695
.500	1.6575	1.4700	.8873
.600	1.5600	1.4117	.9013
.700	1.4855	1.3940	.9120
.800	1.4302	1.3200	.9220
.900	1.3812	1.2930	.9290
1.000	1.3518	1.2800	.9305
1.500	1.2242	1.1730	.9540
2.000	1.1559	1.1227	.9714
5.000	1.0379	1.0300	.9931
10.000	1.0107	1.0087	.9981
25.000	1.0018	1.0015	.9997
50.000	1.0004	1.0004	.9999
100.000	1.0001	1.0001	1.0000
1000.000	1.0000	1.0000	1.0000

$v_{sc}/\bar{v} = \frac{3}{5} = 0.6000$

TABLE A-1 (Cont)

n = 2.5

T	a_1	a_2	J
0	∞	∞	--
.025	6.3882	3.6717	.5740
.050	4.8821	3.1858	.6354
.075	4.2610	2.8728	.6742
.100	3.7976	2.6695	.7028
.200	2.8516	2.2070	.7739
.300	2.4876	1.9625	.8151
.400	2.1368	1.8038	.8434
.500	1.9555	1.6902	.8643
.600	1.8213	1.6039	.8807
.700	1.7182	1.5357	.8938
.800	1.6368	1.4803	.9047
.900	1.5696	1.4342	.9138
1.000	1.5141	1.3953	.9215
1.500	1.3355	1.2696	.9477
2.000	1.2382	1.1926	.9625
5.000	1.0850	1.0544	.9880
10.000	1.0195	1.0185	.9971
25.000	1.0064	1.0029	.9995
50.000	1.0009	1.0007	.9999
100.000	1.0002	1.0002	1.0000
1000.000	1.0000	1.0000	1.0000

$v_{ec}/\bar{v}$ = 0.4408

TABLE A-1 (Cont)

$n = 3.0$

L	a_1	a_2	a_3
0	∞	∞	0
.001	44.1736	12.0559	.2730
.025	10.2779	5.3440	.5200
.050	7.2584	4.3060	.5837
.075	5.8490	3.7703	.6392
.100	5.0892	3.4202	.6721
.200	3.5721	2.6045	.7515
.300	2.9182	2.3244	.7965
.400	2.5391	2.0998	.8270
.500	2.2871	1.9428	.8494
.600	2.1057	1.8255	.8669
.700	1.9681	1.7558	.8809
.800	1.8596	1.6597	.8925
.900	1.7716	1.5966	.9023
1.000	1.6990	1.5471	.9106
1.500	1.4858	1.3760	.9388
2.000	1.3395	1.2793	.9551
5.000	1.1023	1.0878	.9888
10.000	1.0334	1.0291	.9959
25.000	1.0081	1.0054	.9993
50.000	1.0018	1.0014	.9998
100.000	1.0004	1.0003	1.0000
1000.000	1.0000	1.0000	1.0000

$$v_{oc}/\bar{v} = \frac{3\pi}{32} = 0.2945$$

APPENDIX B

AVERAGE ELECTRON-NEUTRAL COLLISION FREQUENCY
FOR WEAKLY IONIZED GASES
(MAXWELLIAN VELOCITY DISTRIBUTION)

For a given i-th neutral species if its electron-neutral (momentum transfer) elastic cross section dependence on energy (or velocity) can be approximated by $Q_{e,n}^{(i)}$

$$Q_{e,n}^{(i)} = q_s{}^{(i)} E_{ev}{}^{s} = q_n{}^{(i)} v_{cm/sec}^{n-1} \quad , \qquad B.1$$

where $q_s{}^{(i)}$ and $q_n{}^{(i)}$ are constants, then one can use Table 6, where for convenience for various n (and s) and various temperatures, the following quantities are listed: $Y_s{}^{(i)}$, $D_s{}^{(i)}$, S_n and $R_s{}^{(i)}$, which are defined by the average (in the sence of Eq (61)) collision frequency of an electron with i-th neutral species (to be used for $\omega >> \nu$, or for any ω with a_1, a_2 and a_3, see Table 5)

$$\bar{\nu}_{e,n}^{(i)}(\sec^{-1}) = Y_s{}^{(i)} q_s{}^{(i)} \frac{p}{p_0} = D_s{}^{(i)} q_s{}^{(i)} N_i(\mathrm{cm}^{-3})$$

$$= S_n C_i q_n{}^{(i)} \frac{p}{p_0} = R_s{}^{(i)} q_s{}^{(i)} P_{mmHg}^{(i)} \quad , \quad \bar{\nu}_{e,n} = \sum_i \bar{\nu}_{e,n}^{(i)} \quad , \qquad B.2$$

where

$$n = 2s + 1 \quad .$$

The quantities were computed from Eq. 180

$P_{mmHg}^{(i)}$ is the (ideal) gas partial pressure in mmHg, N_i is the number density of the i-th neutral species per cm^3, ρ is the mass density of a given gas and ρ_0 is ρ at pressure $P_0 = 1$ atm, and temperature $T_0 = 273.16^\circ K$, C_i for an ideal mixture is the ratio of the number of i-th particles at a given high temperature and pressure (or density) to the total number of particles at P_0 and T_0. If the gas is ideal, nondissociated and nonionized $C_i = X_i$ is the fraction of the i-th particles in the mixture, $\sum_i X_i = 1$. For ideal diatomic gas at T_0 and P_0, which is completely dissociated at some T and P, $\Sigma C_i = 2$.[40] Similarly, for completely ionized ideal gas, which at T_0 and P_0 is monoatomic, $\Sigma C_i = 2$, and for the same gas consisting of only doubly ionized atoms and electrons (e.g., A^{++} and e), $\Sigma C_i = 3$, etc. (The inclusion of real gas corrections, i.e., long distance Coulomb and virial corrections[40] will modify C_i's.) We have

$$N_i = C_i \frac{\rho}{\rho_0} L_0 \quad ; \quad \frac{P^{(i)}}{P_0} = C_i \frac{\rho}{\rho_0} \frac{T}{T_0} \quad , \quad P = \Sigma P^{(i)} \qquad \text{B.3}$$

where P is the total ideal gas pressure and L_0 is the Loschmidt number.

If T_1 is any listed temperature in Table 6, then for any other temperature T

$$\bar{\nu}_{e,n}^{(i)}(T) = \left(\frac{T}{T_1}\right)^{s+1/2} C_i q_s^{(i)} Y_s(T_1) \frac{\rho}{\rho_0} = \left(\frac{T}{T_1}\right)^{s-1/2} q_s^{(i)} R_s(T_1) P_{mmHg}^{(i)} \qquad \text{B.4}$$

The total average electron-neutral collision frequency is

$$\bar{\nu}_{e,n}(T) = \sum_i \bar{\nu}_{e,n}^{(i)} \qquad \text{B.5}$$

For one component gas with complicated velocity (energy) dependence of cross sections we can fit a polynomial to electron neutral cross section (given in E_{ev} or in $v_{cm/sec}$ as in Eq (278)) Then reading directly from Table B-1, $D_s = D_s(t)$ we have

$$\bar{\nu}_{e,n} = N \sum_s q_s D_s(T) \quad . \qquad \text{B.6}$$

There is no direct way of finding q's without additional numerical integration.

For the general case of multicomponent mixture we can fit for the i-th species a polynominal to its cross section, i.e.,

$$Q_{e,n}^{(i)}(\mathrm{cm}^2) = \sum_s q_s{}^{(i)} E_{ev}{}^s = \sum_n q_n{}^{(i)} v^{n-1} \qquad \text{B.7}$$

and reading directly D_s from Table 6

$$\bar{\nu}_{e,n}^{(i)} = N_i \sum_s q_s{}^{(i)} D_s \qquad \text{B.8}$$

with final total average electron collision frequency summed over all neutral constituents,

$$\bar{\nu}_{e,n} = \sum_i \bar{\nu}_{e,n}^{(i)} \qquad \text{B.9}$$

The listed quantities in Table B-1 were computed from, for Maxwellian distribution, and are here summarized:

$$\left.\begin{aligned} Y_s &= \frac{4}{3\sqrt{\pi}} \sqrt{\frac{2kT_e}{m_e}} \left(\frac{T_e}{11605}\right)^s L_o \Gamma(s+3) \\ S_n &= \left(\frac{2k \cdot 11605}{m_e}\right)^s Y_e = \frac{4}{3\sqrt{\pi}} L_o \left(\frac{2kT_e}{m_e}\right)^{n/2} \Gamma\left(\frac{n+5}{2}\right) \\ R_s &= \frac{273.16}{760} \frac{Y_s}{T_e} \quad , \quad D_s = \frac{Y_s}{L_o} \quad , \end{aligned}\right\} \qquad \text{B.10}$$

where $L_0 = 2.6872 \times 10^{19}$ cm^{-3}, and where 1 ev = 11605°K, i.e.,
T is in °K and E in electron volts

$$\frac{1}{2}\frac{m_e v^2}{kT_e} = W = \frac{11605}{T_e} E_{ev} \qquad \text{B-11}$$

Illustrations for the Use of Table B-1. Example 1.

The momentum transfer elastic cross-section for electrons with helium was measured[41] to be*

$$Q \approx 5.3 \times 10^{-16}\ \text{cm}^2 \quad \text{for} \quad 0 \lesssim E < 0.06\ \text{ev} \qquad \text{B-1}$$

Assuming this to be valid for higher energies we obtain an average electron-neutral collision frequency ($\omega >> \nu$) for weakly ionized pure helium at T = 300°K and any pressure (see Table 5).

We have for s 0 at T = 300°K from Table 6, $R_0 = 4.62 \times 10^{23}$

Thus,

$$\bar{\nu}_{e,n} = 4.62 \times 10^{23} \cdot 5.3 \times 10^{-16} P_{mmHg} = 2.5 \times 10^{9} P_{mmHg},\ \text{sec}^{-1}$$

Experimental values for helium for P > 5mmHg was found to be[41]

$$\bar{\nu} \sim 3.12 \times 10^{9} P_{mmHg},\ \text{sec}^{-1} \qquad \text{B-13}$$

Example 2

L.C. Chen[42,43] found that the experimental electron collision frequency with neutral neon atoms at low energies can be fitted with

$$Q_1 = 2.54 \times 10^{-16} \sqrt{E_{ev}}\ ,\ \text{cm}^2 \qquad \text{B-14}$$

or

$$Q_2 = 1.07 \times 10^{-17} + 2.17 \times 10^{-16} \sqrt{E_{ev}}\ ,\ \text{cm}^2 \qquad \text{B-15}$$

---

* See Appendix E for He (Table E-1)

[illegible] for T_e = 100, 200, 300, 400, 500 and 600°K and compare with Chen's experimental values.

From our Table B-1 for n = 2 and n = 1 ($\ell = \frac{1}{2}$, $\ell = 0$) we find

T°K	R_ℓ $\ell = 0$	 $\ell = \frac{1}{2}$
100	8.000×10^{23}	1.234×10^{23}
200	5.657×10^{23}	1.234×10^{23}
300	4.619×10^{23}	1.234×10^{23}
400	4.000×10^{23}	1.234×10^{23}
500	3.578×10^{23}	1.234×10^{23}
600	3.266×10^{23}	1.234×10^{23}

Now, with

$$Q_1(\mathrm{cm}^2) = 2.54 \times 10^{-16}\sqrt{E_{ev}}\ , \quad q_{1/2} = 2.54 \times 10^{-16} \qquad \text{B-16}$$

$$\bar{\nu}_{e,n} = q_{1/2}R_{1/2}P_{mmHg} = 2.54 \times 10^{-16} \cdot (1.234 \times 10^{23})P_{mmHg}$$

we get

T °K	$\bar{\nu}_{e,n}/P \quad \frac{\sec^{-1}}{\mathrm{mmHg}}$
100	3.17×10^7
200	3.17×10^7
300	3.17×10^7
400	3.17×10^7
500	3.17×10^7
600	3.17×10^7

(Note that Chen used P_0 from $P/P_0 = T/273.16$.)

Using the other expression for the cross-section

$$Q_2(\mathrm{cm}^2) = 1.07 \times 10^{-17} + 2.17 \times 10^{-16}\sqrt{E_{ev}}\ , \qquad \text{B-17}$$

we find

$$\bar{\nu}_{e,n} = (q_0R_0 + q_{1/2}R_{1/2})\cdot P_{mmHg},\ q_0 = 1.07 \times 10^{-17},\ q_{1/2} = 2.17 \times 10^{-16} \qquad \text{B-18}$$

and

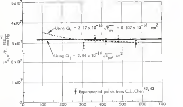

FIGURE B-1 Electron-Neutral Collision Frequency for Neon

T °K	$\bar{\nu}_{e,s}/P$, $\frac{\text{sec}^{-1}}{\text{mmHg}}$
100	3.53×10^7
200	3.28×10^7
300	3.17×10^7
400	3.11×10^7
500	3.06×10^7
600	3.03×10^7

The results are shown in Fig. B-1 and compared with experimental points by Chen.[42,43] Fig. B-1 illustrates the agreement of the fitted cross-sections for this temperature regime.

Example 3.

Given electron-molecule (hypothetical) momentum transfer cross-section dependence on energy as

$$Q = \frac{0.03\times10^{-15}}{E^{3/4}} \quad , \qquad \text{B-19}$$

where Q is cm^2 and E is electron volts. Find for this molecule the electron-neutral collision frequency for very high microwave frequencies, for equilibrium temperature of 300°K and concentration of neutral molecules of 10^{19} cm^{-3} (electrons are assumed to be present from an external source). Answer. Using Table B-1 for $S = -3/4$,

$$\bar{\nu}_{e,n} = 1.26\times10^{8}\cdot(0.03\times10^{-15})\cdot(2\times10^{19}) = 7.56\times10^{10} \text{ sec}^{-1}$$

A L A N POLLI [illegible]

(u [illegible])

$Q_{s,n}^{(t)} \quad q^{(t)} E_{s,s}^{c} - q_n^{(t)} \nu^{-1} \quad | \quad \bar{\nu}_{s,n}^{(t)} \quad n_s q_s^{(t)}$

$n = -3, \; s = -2$

$T, ^\circ K$	τ_s	S_s	n_s	U_s
100	1.6991E+30	1.2113E-01	6.3801E+27	5.5767E+11
200	5.4[illegible]E+29	4.2825E-02	9.5484E+26	1.9724E+11
250	3.7975E+29	3.6443E-02	5.4524E+26	1.4113E+11
300	2.8466E+29	2.5311E-02	3.6805E+26	1.0738E+11
350	2.2494E+29	1.8498E-02	2.3511E+26	8.5199E+10
400	1.8738E+29	1.5141E-02	1.8838E+26	6.9734E+10
450	1.5786E+29	1.2408E-02	1.2643E+26	5.8441E+10
500	1.3455E+29	9.4348E-03	9.9585E+25	4.9808E+10
600	1.[illegible]E+29	8.4218E-03	6.1[illegible]E+25	3.7958E+10
700	8.5664E+28	6.5421E-03	4.1[illegible]E+25	3.0122E+10
800	6.9578E+28	5.3532E-03	2.9765E+25	2.4655E+10
900	5.5524E+28	4.4892E-03	2.2173E+25	2.0682E+10
1000	4.7400E+28	3.8904E-03	1.7689E+25	1.7441E+10
1500	2.5865E+28	2.0850E-03	9.1081E+24	9.8028E+09
2000	1.8701E+28	1.3543E-03	3.8120E+24	6.2372E+09
2500	1.1993E+28	9.8903E-04	1.7942E+24	4.4839E+09
3000	9.1233E+27	7.4717E-04	1.3935E+24	3.3951E+09
3500	7.2499E+27	6.8499E-04	7.4467E+23	2.8942E+09
4000	5.9957E+27	4.7880E-04	5.3249E+23	2.2052E+09
4500	4.9651E+27	4.0124E-04	3.9665E+23	1.8401E+09
5000	4.2401E+27	3.4280E-04	3.0485E+23	1.5778E+09
6000	3.2259E+27	2.9061E-04	1.9322E+23	1.2503E+09
7000	2.5507E+27	2.0662E-04	1.3443E+23	9.5258E+08
8000	2.0951E+27	1.6920E-04	9.4926E+22	7.7906E+08
9000	1.7555E+27	1.4187E-04	7.0310E+22	6.5430E+08
10000	1.4941E+27	1.2135E-04	5.3081E+22	5.5787E+08

$\nu_s^{(t)}(\sec^{-1}) = Y_s^{(t)} q_s^{(t)} \frac{p}{p_s} C \qquad D_s^{(t)} q_s^{(t)} n_s (cm^{-3}) = S_s^{(t)} q_s^{(t)} \frac{p}{p_s} C_1 = R_s^{(t)} q_s^{(t)} \tau_{s}^{(t)} - 3.721 \times 10^{-20} S_s^{(t)} q_s^{(t)} \eta_s$

$\tau_s \; \sum \; \nu^{(t)}$

1 [illegible]

1 [illegible]

[illegible]	[illegible]	[illegible]	K_S	I_S
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	3.9028E+0[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	3.7181E+0[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	5.9508E+[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.6408E+05
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.1450E+04
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.7597E+24	3.6425E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	2.7559E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	2.3947E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.8097E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.3567E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	7.6087E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6.4403E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE [illegible] [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	S
[illegible]	[illegible]	[illegible]	[illegible]	4.5895E+09
[illegible]	[illegible]	[illegible]	[illegible]	2.2946E+09
[illegible]	[illegible]	[illegible]	[illegible]	1.8357E+09
[illegible]	[illegible]	[illegible]	[illegible]	1.5297E+09
[illegible]	[illegible]	[illegible]	[illegible]	1.3112E+09
[illegible]	[illegible]	[illegible]	[illegible]	1.1473E+09
[illegible]	[illegible]	[illegible]	2.1988E+25	1.0198E+09
[illegible]	[illegible]	[illegible]	[illegible]	9.1784E+08
[illegible]	[illegible]	[illegible]	[illegible]	7.6487E+08
[illegible]	[illegible]	[illegible]	9.0457E+24	6.5560E+08
[illegible]	[illegible]	[illegible]	6.9256E+24	5.7365E+08
[illegible]	[illegible]	[illegible]	5.4721E+24	5.0991E+08
100	[illegible]	[illegible]	4.4324E+24	4.5892E+08
150	[illegible]	[illegible]	[illegible]	[illegible]
200	[illegible]	[illegible]	[illegible]	2.2946E+08
250	[illegible]	[illegible]	[illegible]	1.8357E+08
300	[illegible]	[illegible]	[illegible]	1.5297E+08
350	[illegible]	[illegible]	[illegible]	1.3112E+08
400	[illegible]	[illegible]	[illegible]	1.1473E+08
450	[illegible]	[illegible]	2.1988E+23	1.0198E+08
500	[illegible]	[illegible]	[illegible]	9.1784E+07
600	[illegible]	[illegible]	[illegible]	7.6487E+07
700	[illegible]	[illegible]	9.0457E+22	6.5560E+07
800	[illegible]	[illegible]	6.9256E+22	5.7365E+07
900	[illegible]	[illegible]	5.4721E+22	5.0991E+07
1000	[illegible]	[illegible]	4.4324E+22	4.5892E+07

TABLE [illegible]

[illegible]

T, °K	[illegible]	[illegible]	[illegible]	[illegible]
100.	[illegible]	[illegible]	[illegible]	1.4[illegible]
200.	[illegible]	[illegible]	[illegible]	8.6[illegible]
250.	[illegible]	7.2334E+[illegible]	[illegible]	7.29[illegible]
300.	[illegible]	6.4090E+[illegible]	[illegible]	6.36[illegible]
350.	[illegible]	[illegible]	1.5[illegible]E+25	[illegible]
400.	[illegible]	[illegible]	[illegible]	5.1286E+08
450.	[illegible]	4.6847E+28	[illegible]	4.6941E+08
500.	1.1654E+28	4.3010E+28	8.3767E+24	4.3365E+08
600.	[illegible]	3.7513E+28	[illegible]	3.7824E+08
700.	9.[illegible]E+27	3.3418E+28	4.6489E+24	3.3494E+08
800.	8.1912E+27	[illegible]	[illegible]	3.0487E+08
900.	7.6987E+27	2.7477E+28	2.9948E+24	2.7905E+08
1000.	[illegible]	[illegible]	[illegible]	2.5786E+08
1500.	[illegible]	[illegible]	[illegible]	1.9024E+08
2000.	[illegible]	[illegible]	[illegible]	[illegible]
2500.	[illegible]	[illegible]	[illegible]	[illegible]
3000.	[illegible]	[illegible]	[illegible]	1.1312E+08
3500.	[illegible]	[illegible]	[illegible]	[illegible]
4000.	[illegible]	[illegible]	[illegible]	[illegible]
4500.	[illegible]	[illegible]	[illegible]	[illegible]
5000.	[illegible]	7.6484E+27	[illegible]	7.7115E+[illegible]
6000.	[illegible]	[illegible]	[illegible]	[illegible]
7000.	[illegible]	[illegible]	[illegible]	5.9916E+[illegible]
8000.	[illegible]	[illegible]	[illegible]	[illegible]
9000.	[illegible]	[illegible]	[illegible]	[illegible]
10000.	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible] = 1, [illegible] = -1

[illegible]	[illegible]	S_s	N_s	D_s
[illegible]	[illegible]	4.67[illegible]E+12	4.64[illegible]E+2[illegible]	4.8067E+0[illegible]
200	9.[illegible]E+07	[illegible]	[illegible]	3.3069E+0[illegible]
250	8.[illegible]E+07	2.4221E+12	[illegible]	3.0401E+0[illegible]
300	7.[illegible]E+07	2.119[illegible]E+12	8.9[illegible]E+24	2.7752E+0[illegible]
350	6.[illegible]E+07	1.9[illegible]E+12	7.[illegible]E+24	2.4493E+0[illegible]
400	6.[illegible]E+07	1.835[illegible]E+12	5.8[illegible]E+24	2.4034E+0[illegible]
450	6.[illegible]E+07	1.7[illegible]E+12	4.86[illegible]E+24	2.2659E+0[illegible]
500	5.[illegible]E+07	[illegible]420E+12	4.1[illegible]E+24	2.1498E+0[illegible]
600	5.[illegible]E+07	1.4984E+12	[illegible]E+24	1.9623E+0[illegible]
700	4.[illegible]E+07	1.4[illegible]E+12	2.5[illegible]E+24	1.8168E+0[illegible]
800	4.[illegible]E+07	1.2981E+12	2.0[illegible]E+24	1.6904E+0[illegible]
900	4.[illegible]E+07	1.2249E+12	1.7[illegible]E+24	1.6022E+0[illegible]
1000	4.[illegible]E+07	1.[illegible]E+12	1.4601E+24	1.5200E+0[illegible]
1500	3.[illegible]E+07	9.4803E+11	7.99[illegible]E+23	1.2411E+0[illegible]
2000	2.[illegible]E+07	8.2105E+11	5.19[illegible]E+23	[illegible]0748E+0[illegible]
2500	2.[illegible]E+07	7.4482E+11	3.714[illegible]E+23	9.6135E+0[illegible]
3000	2.[illegible]E+07	6.7084E+11	2.8254E+23	8.7759E+0[illegible]
3500	2.[illegible]E+07	6.2962E+11	2.2421E+23	8.1249E+0[illegible]
4000	2.[illegible]E+07	5.8056E+11	1.8457E+23	7.6[illegible]E+0[illegible]
4500	1.[illegible]E+07	5.4735E+11	[illegible]E+23	7.1855E+0[illegible]
5000	1.[illegible]E+07	5.1924E+11	1.3[illegible]E+23	6.[illegible]978E+0[illegible]
6000	1.[illegible]E+07	4.7400E+11	9.9[illegible]E+22	6.2058E+0[illegible]
7000	1.[illegible]E+07	4.4084E+11	7.9[illegible]E+22	5.7492E+0[illegible]
8000	2.[illegible]E+07	4.1150E+11	6.4881E+22	5.3749E+0[illegible]
9000	1.[illegible]E+07	3.8772E+11	5.4174E+22	5.[illegible]E+0[illegible]
10000	1.[illegible]E+07	3.6716E+11	4.[illegible]E+22	4.8[illegible]E+0[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	D_s
[illegible]	[illegible]	[illegible]	[illegible]	1.6593E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.5993E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.3196E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.1392E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.1088E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.0402E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.14[illegible]	9.8480E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	9.[illegible]E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	9.[illegible]E+[illegible]
150	[illegible]	[illegible]	[illegible]	8.4312E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	7.8461E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	7.4204E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	7.0898E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6.8217E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6.[illegible]E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6.4063E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6.2[illegible]E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5.9[illegible]E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5.7564E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-1 (Cont.)

n = 0, s = -1/2

T, °K	τ_s	S_a	η_s	[illegible]
100.	1.5938E+27	2.6872E+19	5.7[illegible]E+24	5.9418E+07
200.	1.5938E+27	2.6872E+19	2.8644E+24	5.9418E+07
250.	1.5938E+27	2.6872E+19	2.2[illegible]4E+24	5.9418E+07
300.	1.5938E+27	2.6872E+19	1.9[illegible]E+24	5.9418E+07
350.	1.5938E+27	2.6872E+19	1.6467E+24	5.9418E+07
400.	1.5938E+27	2.6872E+19	1.4[illegible]E+24	5.9418E+07
450.	1.5938E+27	2.6872E+19	1.27[illegible]E+24	5.9418E+07
500.	1.5938E+27	2.6872E+19	1.1457E+24	5.9418E+07
600.	1.5938E+27	2.6872E+19	9.5427E+23	5.9418E+07
700.	1.5938E+27	2.6872E+19	8.1847E+23	5.9418E+07
800.	1.5938E+27	2.6872E+19	7.1407E+23	5.9418E+07
900.	1.5938E+27	2.6872E+19	6.3451E+23	5.9418E+07
1000.	1.5938E+27	2.6872E+19	5.72[illegible]E+23	5.9418E+07
1500.	1.5938E+27	2.6872E+19	3.8191E+23	5.9418E+07
2000.	1.5938E+27	2.6872E+19	2.8448E+23	5.9418E+07
2500.	1.5938E+27	2.6872E+19	2.2034E+23	5.9418E+07
3000.	1.5938E+27	2.6872E+19	1.9295E+23	5.9418E+07
3500.	1.5938E+27	2.6872E+19	1.6467E+23	5.9418E+07
4000.	1.5938E+27	2.6872E+19	1.4[illegible]E+23	5.9418E+07
4500.	1.5938E+27	2.6872E+19	1.27[illegible]E+23	5.9418E+07
5000.	1.5938E+27	2.6872E+19	1.1457E+23	5.9418E+07
6000.	1.5938E+27	2.6872E+19	9.5427E+22	5.9418E+07
7000.	1.5938E+27	2.6872E+19	8.1847E+22	5.9418E+07
8000.	1.5938E+27	2.6872E+19	7.1407E+22	5.9418E+07
9000.	1.5938E+27	2.6872E+19	6.3451E+22	5.9418E+07
10000.	1.5938E+27	2.6872E+19	5.7[illegible]E+22	5.9418E+07

TABLE B-[illegible]

n = 1/2, s = -1/4

T, °K	Y_s	S_s	R_s	T_s
10	3.4792E+06	7.6287E+22	2.1[illegible]E+24	2.1844E+07
20	4.4803E+06	9.0720E+22	1.2454E+24	2.6006E+07
25	7.0670E+06	9.5925E+22	1.0621E+24	2.7497E+07
30	7.7666E+06	1.0041E+23	9.2637E+23	2.8774E+07
35	8.3301E+06	1.0434E+23	8.2573E+23	2.9905E+07
40	8.6605E+06	1.0789E+23	7.4459E+23	3.0927E+07
45	8.5570E+06	1.1111E+23	6.8545E+23	3.1844E+07
50	8.7254E+06	1.1407E+23	6.3[illegible]E+23	3.2694E+07
60	9.1951E+06	1.1939E+23	5.5182E+23	3.4218E+07
70	9.5464E+06	1.2409E+23	4.9168E+23	3.5548E+07
80	9.8818E+06	1.2830E+23	4.4392E+23	3.6778E+07
90	1.0170E+07	1.3214E+23	4.0649E+23	3.7869E+07
100	1.0489E+07	1.3566E+23	3.7551E+23	3.8880E+07
150	1.1486E+07	1.5013E+23	2.7705E+23	4.3027E+07
200	1.2044E+07	1.6135E+23	2.2326E+23	4.6236E+07
250	1.3157E+07	1.7058E+23	1.8867E+23	4.8849E+07
300	1.3750E+07	1.7854E+23	1.6474E+23	5.1149E+07
350	1.4290E+07	1.8555E+23	1.4675E+23	5.3179E+07
400	1.4775E+07	1.9185E+23	1.3276E+23	5.4984E+07
450	1.5217E+07	1.9758E+23	1.2154E+23	5.6627E+07
500	1.5625E+07	2.0284E+23	1.1234E+23	5.8139E+07
600	1.6352E+07	2.1232E+23	9.7951E+22	6.0850E+07
700	1.6994E+07	2.2068E+23	8.7257E+22	6.3241E+07
800	1.7571E+07	2.2815E+23	7.8442E+22	6.5388E+07
900	1.8096E+07	2.3497E+23	7.2267E+22	6.7342E+07
1000	1.8579E+07	2.4124E+23	6.6777E+22	6.9139E+07

TABLE 5-3

n = 1, s = 0

T, °K	T_s	S_s	m_s	t_s
100	2.2259E+26	[illegible]	8.66[illegible]	8.2[illegible]
20	[illegible]	3.3478E+2[illegible]	5.6570E+2[illegible]	1.1714E+[illegible]
25	[illegible]	[illegible]	5.0908E+2[illegible]	1.3097E+[illegible]
30	[illegible]	3.2553E+2[illegible]	4.6[illegible]	1.4447E+[illegible]
35	[illegible]	[illegible]	4.2748E+2[illegible]	1.5408E+[illegible]
40	[illegible]	[illegible]	4.2[illegible]	1.6588E+[illegible]
45	[illegible]	[illegible]	[illegible]	1.7471E+[illegible]
50	[illegible]	4.9772E+2[illegible]	3.5778E+2[illegible]	1.8522E+[illegible]
60	[illegible]	[illegible]	3.2661E+2[illegible]	2.0290E+[illegible]
70	5.8[illegible]	5.8881E+2[illegible]	[illegible]	2.1919E+[illegible]
80	[illegible]	[illegible]	2.8285E+2[illegible]	2.3428E+[illegible]
90	[illegible]	[illegible]	2.6667E+2[illegible]	2.4850E+[illegible]
100	[illegible]	[illegible]	2.5298E+2[illegible]	2.6194E+[illegible]
150	[illegible]	8.6217E+2[illegible]	[illegible]	3.2081E+[illegible]
200	[illegible]	9.9543E+2[illegible]	[illegible]	3.7044E+[illegible]
250	[illegible]	1.1129E+2[illegible]	[illegible]	4.1416E+[illegible]
300	[illegible]	[illegible]	[illegible]	4.5465E+[illegible]
350	[illegible]	[illegible]	[illegible]	[illegible]
400	[illegible]	[illegible]	[illegible]	5.2480E+[illegible]
450	[illegible]	[illegible]	[illegible]	5.5586E+[illegible]
500	[illegible]	[illegible]	[illegible]	5.8521E+[illegible]
600	[illegible]	[illegible]	[illegible]	6.4162E+[illegible]
700	[illegible]	[illegible]	[illegible]	6.9302E+[illegible]
800	[illegible]	[illegible]	[illegible]	7.4087E+[illegible]
900	[illegible]	[illegible]	[illegible]	7.8581E+[illegible]
1000	[illegible]	[illegible]	[illegible]	8.2856E+[illegible]

TABLE B-1 (Cont)

n = 3/2, s = 1/4

T, °K	r_S	S_s	[illegible]	r_s
150	8.8659E+25	8.6571E+29	3.1764E+23	4.2167E+06
200	1.4557E+26	1.1196E+30	2.61[illegible]E+23	5.4098E+06
250	[illegible].7806E+26	1.3235E+30	2.4728E+23	6.3954E+06
300	[illegible].9746E+26	1.5175E+30	2.3607E+23	7.3328E+06
350	2.2119E+26	1.7035E+30	2.2[illegible]4E+23	8.2312E+06
400	2.4249E+26	1.8829E+30	2.1568E+23	9.0042E+06
450	2.6777E+26	2.0564E+30	2.1351E+23	9.9485E+06
500	2.8035E+26	2.2250E+30	2.0774E+23	1.0756E+07
600	3.3150E+26	2.5521E+30	1.9851E+23	1.2552E+07
700	3.7199E+26	2.8649E+30	1.9[illegible]E+23	1.3843E+07
800	4.1118E+26	3.1687E+30	1.8475E+23	1.5303E+07
900	4.4815E+26	3.4591E+30	1.7947E+23	1.6715E+07
1000	4.8808E+26	3.7435E+30	1.7421E+23	1.8089E+07
1500	6.5884E+26	5.0749E+30	1.5787E+23	2.4516E+07
2000	8.0749E+26	6.2959E+30	1.4601E+23	3.0422E+07
2500	9.6642E+26	7.4428E+30	1.3894E+23	3.5964E+07
3000	1.1[illegible]0E+27	8.5384E+30	1.3275E+23	4.1234E+07
3500	1.2830E+27	9.5793E+30	1.2773E+23	4.6287E+07
4000	1.4749E+27	1.0588E+31	1.2354E+23	5.1183E+07
4500	1.6019E+27	1.1560E+31	1.1995E+23	5.6088E+07
5000	1.6353E+27	1.2517E+31	1.1683E+23	6.0464E+07
6000	1.8859E+27	1.4351E+31	1.1146E+23	6.9447E+07
7000	2.0195E+27	1.6110E+31	1.0741E+23	7.7846E+07
8000	2.3172E+27	1.7807E+31	1.0388E+23	8.6046E+07
9000	2.5250E+27	1.9452E+31	[illegible]87E+23	9.4098E+07
10000	2.7335E+27	2.1052E+31	9.8246E+22	1.0172E+08

TABLE B-1 (Cont)

[illegible] − 2. [illegible]

T, °K	r_0	r_1	[illegible]	[illegible]
1.0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.5	[illegible]	[illegible]	[illegible]	3.1941E+[illegible]
3.0	[illegible]	[illegible]	[illegible]	3.8329E+[illegible]
[illegible]	[illegible]	[illegible]	1.2340E+23	4.4717E+[illegible]
[illegible]	[illegible]	[illegible]	1.2340E+23	5.1106E+[illegible]
[illegible]	[illegible]	[illegible]	1.2340E+23	5.7493E+[illegible]
[illegible]	[illegible]	[illegible]	1.2340E+23	6.3881E+[illegible]
[illegible]	[illegible]	[illegible]	1.2340E+23	7.6658E+[illegible]
7.0	[illegible]	[illegible]	1.2340E+23	8.9434E+[illegible]
8.0	[illegible]	1.4254E+[illegible]	1.2340E+23	1.0221E+[illegible]
9.0	[illegible]	1.6291E+[illegible]	1.2340E+23	1.1499E+[illegible]
10.0	[illegible]	1.8327E+[illegible]	1.2340E+23	1.2778E+[illegible]
15.0	[illegible]	[illegible]	1.2340E+23	1.9166E+[illegible]
20.0	[illegible]	[illegible]	1.2340E+23	2.5553E+[illegible]
25.0	[illegible]	[illegible]	1.2340E+23	3.1941E+[illegible]
30.0	[illegible]	[illegible]	1.2340E+23	3.8329E+[illegible]
35.0	[illegible]	[illegible]	1.2340E+23	4.4717E+[illegible]
40.0	[illegible]	[illegible]	1.2340E+23	[illegible]
45.0	[illegible]	[illegible]	1.2340E+23	[illegible]
50.0	[illegible]	[illegible]	1.2340E+23	[illegible]
60.0	[illegible]	[illegible]	1.2340E+23	[illegible]
70.0	[illegible]	[illegible]	1.2340E+23	[illegible]
80.0	[illegible]	[illegible]	1.2340E+23	[illegible]
90.0	[illegible]	[illegible]	[illegible]	[illegible]
100.00	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-1 (Cont)

$n = 5/2, \varepsilon = 3/4$

T, °K	Y_s	S_s	W_s	I_s
1	1.4[illegible]E+05	5.4598E+36	5.[illegible]E+22	5.1805E+05
2	3.[illegible]E+05	[illegible].9124E+37	5.9[illegible]E+22	1.2321E+06
25	4.[illegible]E+05	1.1990E+37	6.2[illegible]E+22	1.6285E+06
30	5.4265E+05	[illegible].4187E+37	6.585[illegible]E+22	2.0454E+06
35	6.6643E+05	3.0442E+37	6.8447E+22	2.4800E+06
40	7.6724E+05	3.4972E+37	7.0[illegible]E+22	2.9369E+06
45	9.[illegible]E+05	4.1278E+37	7.2[illegible]E+22	3.3954E+06
50	1.[illegible]E+06	4.7545E+37	7.4[illegible]E+22	3.8733E+06
60	1.[illegible]E+06	5.9714E+37	7.8[illegible]E+22	4.8648E+06
70	1.5[illegible]E+06	7.2434E+37	8.[illegible]E+22	5.8985E+06
80	1.[illegible]E+06	8.5557E+37	8.4[illegible]E+22	6.9709E+06
90	2.1[illegible]E+06	9.9127E+37	8.6[illegible]E+22	8.0756E+06
100	2.4[illegible]E+06	1.1388E+38	8.8976E+22	9.2124E+06
150	4.1[illegible]E+06	[illegible].8722E+38	9.6469E+22	1.5293E+07
200	5.8[illegible]E+06	2.6895E+38	1.0581E+23	2.1911E+07
250	7.7[illegible]E+06	3.5548E+38	1.1[illegible]E+23	2.8960E+07
300	9.7[illegible]E+06	4.4647E+38	1.1[illegible]E+23	3.6173E+07
350	1.1951E+07	5.4135E+38	1.2171E+23	4.4102E+07
400	1.4[illegible]E+07	6.3988E+38	1.2584E+23	5.2113E+07
450	1.6725E+07	7.4115E+38	1.2959E+23	6.0379E+07
500	1.9[illegible]E+07	8.4545E+38	1.3[illegible]E+23	6.8879E+07
600	2.4[illegible]E+07	1.0619E+39	1.3926E+23	8.6509E+07
700	2.9[illegible]E+07	[illegible].2875E+39	1.4473E+23	1.0489E+08
800	3.4[illegible]E+07	1.5214E+39	1.4944E+23	1.2395E+08
900	3.9[illegible]E+07	[illegible].7628E+39	1.5411E+23	1.4361E+08
1000	4.[illegible]E+07	[illegible].0109E+[illegible]	1.5[illegible]E+23	1.6382E+08

TABLE B-1 [illegible]

[illegible] 1

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6.8560E+0[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	8.4635E+[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.1125E+06
15	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.7129E+06
[illegible]	[illegible]	[illegible]	[illegible]	2.0439E+06
[illegible]	[illegible]	2.2830E+4[illegible]	4.6741E+22	[illegible]
[illegible]	[illegible]	2.9748E+4[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	3.9654E+06
[illegible]	[illegible]	4.5800E+41	[illegible]	4.8448E+06
[illegible]	[illegible]	[illegible]	[illegible]	5.7810F+06
[illegible]	[illegible]	[illegible]	[illegible]	6.7708E+06
[illegible]	[illegible]	1.1750E+42	[illegible]	1.2439E+07
200	[illegible]	1.8104E+42	[illegible]	1.9151E+07
250	[illegible]	2.5401E+42	[illegible]	2.6764E+07
300	[illegible]	3.3259E+42	[illegible]	[illegible]
350	1.1913E+27	4.1911E+42	[illegible]	4.4444E+07
400	1.4555E+27	5.1208E+42	[illegible]	5.4166E+07
450	1.7348E+27	6.1101E+42	[illegible]	[illegible]
500	[illegible]	7.1562E+42	[illegible]	7.5499E+07
600	2.6741E+27	9.4071E+42	[illegible]	[illegible]
700	[illegible]	1.1854E+43	[illegible]	1.2640E+08
800	[illegible]	1.4483E+43	1.8498E+23	[illegible]
900	[illegible]	1.7282E+43	[illegible]	1.8981E+08
1000	[illegible]	2.0241E+43	[illegible]	2.1411E+08

TABLE B-1 (Cont)

$n = 4, s = 3/2$

T °K	Y_s	S_s	H_s	
100	1.0354E+24	2.1601E+47	3.7216E+21	3.8533E+04
150	2.3297E+24	4.8603E+47	5.5823E+21	8.6700E+04
200	4.1417E+24	8.6404E+47	7.4431E+21	1.5413E+05
250	6.4715E+24	1.3501E+48	9.3039E+21	2.4083E+05
300	9.3189E+24	1.9441E+48	1.1165E+22	3.4680E+05
350	1.2684E+25	2.6462E+48	1.3025E+22	4.7204E+05
400	1.6567E+25	3.4562E+48	1.4886E+22	6.1654E+05
450	2.0968E+25	4.3743E+48	1.6747E+22	7.8030E+05
500	2.5886E+25	5.4004E+48	1.8608E+22	9.6334E+05
600	3.7276E+25	7.7764E+48	2.2329E+22	1.3872E+06
700	5.0736E+25	1.0585E+49	2.6051E+22	1.8881E+06
800	6.6268E+25	1.3825E+49	2.9773E+22	2.4661E+06
900	8.3870E+25	1.7497E+49	3.3494E+22	3.1212E+06
1000	1.0354E+26	2.1601E+49	3.7216E+22	3.8533E+06
1500	2.3297E+26	4.8603E+49	5.5823E+22	8.6700E+06
2000	4.1417E+26	8.6404E+49	7.4431E+22	1.5413E+07
2500	6.4715E+26	1.3501E+50	9.3039E+22	2.4083E+07
3000	9.3189E+26	1.9441E+50	1.1165E+23	3.4680E+07
3500	1.2684E+27	2.6462E+50	1.3025E+23	4.7204E+07
4000	1.6567E+27	3.4562E+50	1.4886E+23	6.1654E+07
4500	2.0968E+27	4.3743E+50	1.6747E+23	7.8030E+07
5000	2.5886E+27	5.4004E+50	1.8608E+23	9.6334E+07
6000	3.7276E+27	7.7764E+50	2.2329E+23	1.3872E+08
7000	5.0736E+27	1.0585E+51	2.6051E+23	1.8881E+08
8000	6.6268E+27	1.3825E+51	2.9773E+23	2.4661E+08
9000	8.3870E+27	1.7497E+51	3.3494E+23	3.1212E+08
10000	1.0354E+28	2.1601E+51	3.7216E+23	3.8533E+08

Table B-2 (Cont)

$n = 5,\ s = 2$

$T^{o}K$	Y_s	S_s	R_s	D_s
100	1.6832E+23	2.4536E+54	7.1280E+20	7.3804E+03
150	5.4651E+23	6.7620E+54	1.3069E+21	2.0736E+04
200	1.1210E+24	1.3081E+55	2.0161E+21	4.1750E+04
250	1.9596E+24	2.4249E+55	2.8176E+21	7.2634E+04
300	3.1615E+24	3.6252E+55	3.7036E+21	1.1505E+05
350	4.5450E+24	5.6237E+55	4.6674E+21	1.6614E+05
400	6.3462E+24	7.6523E+55	5.7024E+21	2.3617E+05
450	8.5182E+24	1.0541E+56	6.8044E+21	3.1704E+05
500	1.1086E+25	1.3717E+56	7.6694E+21	4.1756E+05
600	1.7486E+25	2.1638E+56	1.0476E+22	6.5082E+05
700	2.5711E+25	3.1612E+56	1.3201E+22	6.9681E+05
800	3.5600E+25	4.4420E+56	1.6126E+22	1.3360E+06
900	4.8162E+25	5.6628E+56	1.9246E+22	1.7634E+06
1000	6.2714E+25	7.7598E+56	2.2541E+22	2.3336E+06
1500	1.7282E+26	2.1387E+57	4.1410E+22	6.4315E+06
2000	3.5477E+26	4.3866E+57	6.3755E+22	1.3203E+07
2500	6.1675E+26	7.6683E+57	8.6100E+22	2.3064E+07
3000	9.7762E+26	1.2066E+58	1.1713E+23	3.6382E+07
3500	1.4376E+27	1.7784E+58	1.4760E+23	5.3488E+07
4000	2.0069E+27	2.4831E+58	1.8033E+23	7.4685E+07
4500	2.6640E+27	3.3333E+58	2.1517E+23	1.0025E+08
5000	3.5050E+27	4.3378E+58	2.5201E+23	1.3047E+08
6000	5.5302E+27	6.8427E+58	3.3128E+23	2.0581E+08
7000	8.1304E+27	1.0060E+59	4.1746E+23	3.0257E+08
8000	1.1354E+28	1.4047E+59	5.1064E+23	4.2248E+08
9000	1.5240E+28	1.8856E+59	6.0860E+23	5.6714E+08
10000	1.9832E+28	2.4534E+59	7.1280E+23	7.3804E+08

Table 8-2 (Cont)

$n = 6, s = 5/2$

$T\,{}^{\circ}K$	Y_s	S_s	N_s	P_s
100	4.0150E+22	2.9464E+61	1.4431E+20	1.4942E+03
150	1.3551E+23	9.9441E+61	3.2469E+20	5.0429E+03
200	3.2120E+23	2.3571E+62	5.7724E+20	1.1954E+04
250	6.2733E+23	4.6038E+62	9.0193E+20	2.3347E+04
300	1.0841E+24	7.9553E+62	1.2988E+21	4.0343E+04
350	1.7214E+24	1.2633E+63	1.7678E+21	6.4063E+04
400	2.5696E+24	1.8857E+63	2.3089E+21	9.5628E+04
450	3.6587E+24	2.6849E+63	2.9223E+21	1.3616E+05
500	5.0188E+24	3.6830E+63	3.6077E+21	1.8677E+05
600	8.6725E+24	6.3642E+63	5.1951E+21	3.2274E+05
700	1.3772E+25	1.0106E+64	7.0711E+21	5.1251E+05
800	2.0557E+25	1.5086E+64	9.2358E+21	7.6503E+05
900	2.9270E+25	2.1479E+64	1.1689E+22	1.0893E+06
1000	4.0150E+25	2.9464E+64	1.4431E+22	1.4942E+06
1500	1.3551E+26	9.9441E+64	3.2469E+22	5.0429E+06
2000	3.2120E+26	2.3571E+65	5.7724E+22	1.1954E+07
2500	6.2733E+26	4.6038E+65	9.0193E+22	2.3347E+07
3000	1.0841E+27	7.9553E+65	1.2988E+23	4.0343E+07
3500	1.7214E+27	1.2633E+66	1.7678E+23	6.4063E+07
4000	2.5696E+27	1.8857E+66	2.3089E+23	9.5628E+07
4500	3.6587E+27	2.6849E+66	2.9223E+23	1.3616E+08
5000	5.0188E+27	3.6830E+66	3.6077E+23	1.8677E+08
6000	8.6725E+27	6.3642E+66	5.1951E+23	3.2274E+08
7000	1.3772E+28	1.0106E+67	7.0711E+23	5.1251E+08
8000	2.0557E+28	1.5086E+67	9.2358E+23	7.6503E+08
9000	2.9270E+28	2.1479E+67	1.1689E+24	1.0893E+09
10000	4.0150E+28	2.9464E+67	1.4431E+24	1.4942E+09

T °K	r_s	S_s	H_s	[illegible]
100	8.5440E+21	3.7189E+68	3.0711E+19	3.1799E+02
150	1.5319E+22	1.5372E+69	8.4430E+19	1.3144E+03
200	9.6671E+22	4.2075E+69	1.7373E+20	3.5979E+03
250	2.1110E+23	9.1677E+69	3.0349E+20	7.8559E+03
300	1.9959E+23	1.7392E+70	4.7874E+20	1.4871E+04
350	6.4550E+23	2.9630E+70	7.0383E+20	2.5506E+04
400	1.0937E+24	4.7602E+70	9.8275E+20	4.0702E+04
450	1.6517E+24	7.1888E+70	1.3192E+21	6.1468E+04
500	2.3893E+24	1.0385E+71	1.7168E+21	8.8800E+04
600	4.5209E+24	1.9678E+71	2.7062E+21	1.6824E+05
700	7.7542E+24	3.3749E+71	3.9814E+21	2.8857E+05
800	1.2374E+25	5.3855E+71	5.5593E+21	4.6049E+05
900	1.8687E+25	8.1332E+71	7.4028E+21	6.9543E+05
1000	2.7020E+25	1.1760E+72	9.7117E+21	1.0056E+06
1500	1.1169E+26	4.8611E+72	2.6702E+22	4.1565E+06
2000	3.0570E+26	1.3305E+73	5.4938E+22	1.1377E+07
2500	6.6755E+26	2.9054E+73	9.5972E+22	2.4843E+07
3000	1.2630E+27	5.4497E+73	1.5139E+23	4.7025E+07
3500	2.1674E+27	9.4331E+73	2.2257E+23	8.0650E+07
4000	3.4589E+27	1.5053E+74	3.1077E+23	1.2871E+08
4500	5.2232E+27	2.2733E+74	4.1718E+23	1.9458E+08
5000	7.5524E+27	3.2871E+74	5.4290E+23	2.8106E+08
6000	1.4299E+28	6.2227E+74	8.5639E+23	5.3206E+08
7000	2.4521E+28	1.0672E+75	1.2590E+24	9.1254E+08
8000	3.9133E+28	1.7031E+75	1.7560E+24	1.4562E+09
9000	5.9094E+28	2.5720E+75	2.3599E+24	2.1992E+09
10000	8.5440E+28	3.7189E+75	3.0711E+24	3.1799E+09

Table B-1 (Cont)

n = 8, s = 7/2

T °K	I_s	S_s	W_s	J
100	1.9029E+21	4.9119E+75	6.8393E+18	7.0815E+01
150	9.6332E+21	2.4867E+76	2.3083E+19	3.5850E+02
200	3.0446E+22	7.8591E+76	5.4714E+19	1.1330E+03
250	7.4331E+22	1.9187E+77	1.0686E+20	2.7662E+03
300	1.5413E+23	3.9787E+77	1.8466E+20	5.7360E+03
350	2.8555E+23	7.3709E+77	2.9323E+20	1.0627E+04
400	4.8713E+23	1.2575E+78	4.3771E+20	1.8129E+04
450	7.8029E+23	2.0142E+78	6.2323E+20	2.9038E+04
500	1.1893E+24	3.0699E+78	8.5491E+20	4.4259E+04
600	2.4661E+24	6.3658E+78	1.4773E+21	9.1776E+04
700	4.5688E+24	1.1794E+79	2.3459E+21	1.7003E+05
800	7.7941E+24	2.0119E+79	3.5017E+21	2.9006E+05
900	1.2485E+25	3.2227E+79	4.9858E+21	4.6462E+05
1000	1.9029E+25	4.9119E+79	6.8393E+21	7.0815E+05
1500	9.6332E+25	2.4867E+80	2.3083E+22	3.5850E+06
2000	3.0446E+26	7.8591E+80	5.4714E+22	1.1330E+07
2500	7.4331E+26	1.9187E+81	1.0686E+23	2.7662E+07
3000	1.5413E+27	3.9787E+81	1.8466E+23	5.7360E+07
3500	2.8555E+27	7.3709E+81	2.9323E+23	1.0627E+08
4000	4.8713E+27	1.2575E+82	4.3771E+23	1.8129E+08
4500	7.8029E+27	2.0142E+82	6.2323E+23	2.9038E+08
5000	1.1893E+28	3.0699E+82	8.5491E+23	4.4259E+08
6000	2.4661E+28	6.3658E+82	1.4773E+24	9.1776E+08
7000	4.5688E+28	1.1794E+83	2.3459E+24	1.7003E+09
8000	7.7941E+28	2.0119E+83	3.5017E+24	2.9006E+09
9000	1.2485E+29	3.2227E+83	4.9858E+24	4.6462E+09
10000	1.9029E+29	4.9119E+83	6.8393E+24	7.0815E+09

Table B-1 (Cont)

n = 9, s = 4

T °K	Y_s	S_s	R_s	
100	4.4177E+20	6.7634E+02	1.5878E+18	1.6440E+01
150	2.7391E+21	4.1935E+03	6.5633E+18	1.0194E+02
200	9.9961E+21	1.5304E+04	1.7964E+19	3.7200E+02
250	2.7249E+22	4.1773E+04	3.9228E+19	1.0154E+03
300	6.1979E+22	9.4867E+04	7.4255E+19	2.3065E+03
350	1.2402E+23	1.8968E+05	1.2736E+20	4.6155E+03
400	2.2619E+23	3.4628E+05	2.0324E+20	8.4175E+03
450	3.8429E+23	5.8831E+05	3.0693E+20	1.4301E+04
500	6.1739E+23	9.4521E+05	4.4381E+20	2.2976E+04
600	1.4024E+24	2.1471E+06	8.4010E+20	5.2191E+04
700	2.8063E+24	4.2964E+06	1.4409E+21	1.0444E+05
800	5.1168E+24	7.8335E+06	2.2994E+21	1.9047E+05
900	8.6954E+24	1.3312E+07	3.4726E+21	3.2360E+05
1000	1.3970E+25	2.1388E+07	5.0211E+21	5.1989E+05
1500	8.6619E+25	1.3261E+08	2.0755E+22	3.2235E+06
2000	3.1611E+26	4.8395E+08	5.6808E+22	1.1764E+07
2500	8.6283E+26	1.3210E+09	1.2405E+23	3.2110E+07
3000	1.9599E+27	3.0006E+09	2.3481E+23	7.2939E+07
3500	3.9220E+27	6.0044E+09	4.0275E+23	1.4596E+08
4000	7.1527E+27	1.0950E+10	6.4270E+23	2.6619E+08
4500	1.2152E+28	1.8605E+10	9.7061E+23	4.5224E+08
5000	1.9524E+28	2.9690E+10	1.4034E+24	7.2657E+08
6000	4.4349E+28	6.7498E+10	2.6566E+24	1.6504E+09
7000	8.8744E+28	1.3598E+11	4.5566E+24	3.3026E+09
8000	1.6180E+29	2.4778E+11	7.2714E+24	6.0231E+09
9000	2.7497E+29	4.2097E+11	1.0981E+25	1.0233E+10
10000	4.4177E+29	6.7634E+11	1.5878E+25	1.6440E+10

APPENDIX C

CORRECTIONS α_1, α_2 and α_3 FOR THE ELECTRIC CONDUCTIVITY FOR WEAKLY IONIZED N_2, O_2 AND AIR

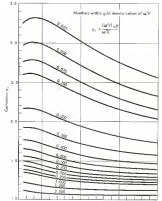

FIGURE C-1 Correction α_1 for Weakly Ionized Nitrogen, N_2

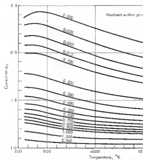

FIGURE C-2 Correction α_e for Weakly Ionized Nitrogen, N_2

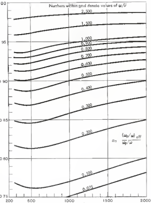

FIGURE C-3 Correction α_3 for Weakly Ionized Nitrogen, N_2

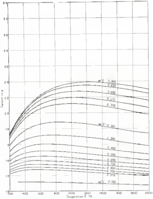

FIGURE C.4 Correction η for Weakly Ionized Oxygen O at Constant ω/p (MODEL I)

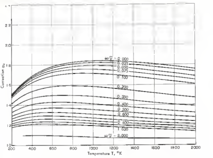

FIGURE C-5 Correction α_2 for Weakly Ionized Oxygen, O_2, at Constant $\omega/\bar{\nu}$ (MODEL 1)

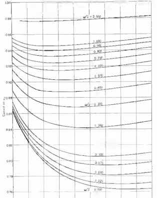

FIGURE C-6 Correction α_2 for Weakly Ionized Oxygen [illegible] Constant $\omega/\overline{\nu}$ (MODEL I)

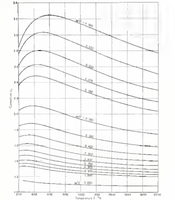

FIGURE C 7 Correction α₁ for Weakly Ionized Air (Model I)

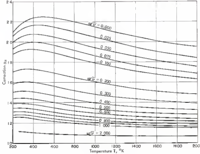

FIGURE C-8 Correction α_s for Weakly Ionized Air (Model I)

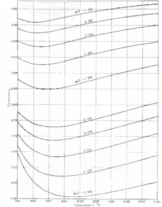

FIGURE C-9 Correction φ for Weakly Ionized Air (Model I)

APPENDIX D

AVERAGE ELECTRON COLLISION FREQUENCIES FOR WEAKLY IONIZED N_2, O_2, NO, N, O AND DRY AIR

($\omega >> \nu$)

For $\omega >> \nu$, we have from Eq. 29

$$\bar{\nu} = \frac{4}{3\sqrt{\pi}} \int_0^\infty \nu(W)\, W\sqrt{W}\, e^{-W} dW \quad , \quad W = \frac{1}{2}\frac{mv^2}{kT}$$

$$\varepsilon_{ev} = W \frac{T\,10^{-4}}{1.1606}$$

where

$$N(\mathrm{cm}^{-3}) = L_0\, \rho/\rho_0 \quad , \quad L_0 = 2.6871 \times 10^{19} \quad ,$$

$$T_0 = 273.16°\mathrm{K} \quad , \quad P_0 = 1 \text{ atm},$$

$$P_{mm\,Hg} = 760(\rho/\rho_0)T/273.16 \quad .$$

$\rho_0(N_2) = 1.250 \times 10^{-3}$ gm/cm^3 , ρ_0(dry air) $= 1.223 \times 10^{-3}$ gm/cm^3

$\rho_0(O_2) = 1.428 \times 10^{-3}$ gm/cm^3 ; ρ_0(h.temp.air) $= 1.293 \times 10^{-3}$ gm/cm^3

Dry air: 78.0881% N_2 , 20.9795% O_2 , 0.9324% A.

High temp. air: 78.084% N_2 ; 20.946% O_2, 0.937% A, 0.033% CO_2
(only concentrations of N_2, O_2, N, O, NO and electrons were used)

For i-th species,

$$\bar{\nu}(i) = \bar{\nu}_0^{(i)} \cdot 10^{11}\, C_i \frac{\rho}{\rho_0} = \bar{\nu}_1^{(i)}\, 10^{8}\, P^{(i)}_{mm\,Hg} = \bar{\nu}_2^{(i)}\, 10^{-8}\, N^{(i)}_{cm^{-3}} \quad ,$$

where $\bar{\nu}_0$, $\bar{\nu}_1$ and $\bar{\nu}_2$ are factors tabulated in the next tables and where

$$\bar{\nu}_2^{(i)} = \bar{\nu}_0^{(i)}/2.6871 \quad .$$

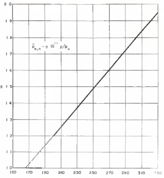

FIGURE D-1 Average Collision Frequency for Weakly Ionized Dry Low Temperature Air

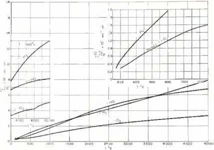

FIGURE D-2 Average Electron Collision Frequency for Weakly Ionized N_2, O_2 and NO ($\omega \gg \nu$)

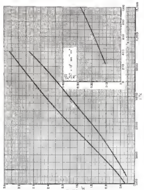

FIGURE D-3 Average Electron Collision Frequency for Atomic Oxygen and Atomic Nitrogen $(\bar{\nu}_{e-n}^{(1)})$

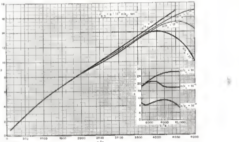

FIGURE D-4 Average Electron-Neutral Collision Frequency in Equilibrium Air ($\omega \gg \nu$)

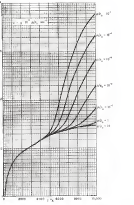

FIGURE D-5 Total Average Electron Collision Frequency of Equilibrium Air ($\omega >> \nu$)
(Electron-Neutral + Electron-Ion)

TABLE D-1

AVERAGE ELECTRON COLLISION FREQUENCY FOR HIGH TEMPERATURE EQUILIBRIUM DRY AIR*

$(\omega \gg \nu, \quad \bar{\nu} = \bar{\nu}_{e,n} + \bar{\nu}_{e,i})$

T, °K	$\log_{10}\rho/\rho_0$	$\bar{\nu}_{e,n}$, sec^{-1}	$\bar{\nu}$, sec^{-1}	$10^{-[illegible]}\ \bar{\nu}/\frac{\rho}{\rho_0}$
[illegible]	-5	8.459×10^{6}	8.459×10^{6}	8.459
	-4	8.484×10^{7}	8.484×10^{7}	8.484
	-3	8.492×10^{8}	8.492×10^{8}	8.492
	-2	8.495×10^{9}	8.495×10^{9}	8.495
	-1	8.495×10^{10}	8.495×10^{10}	8.495
[illegible]	-5	9.746×10^{6}	9.750×10^{6}	9.750
	-4	9.924×10^{7}	9.925×10^{7}	9.925
	-3	10.03×10^{8}	10.04×10^{8}	10.04
	-2	1.008×10^{10}	1.008×10^{10}	10.08
	-1	1.009×10^{11}	1.009×10^{11}	10.09
	0	1.010×10^{12}	1.010×10^{12}	10.10
	1	1.010×10^{13}	1.010×10^{13}	10.10
5000	-5	1.127×10^{7}	1.132×10^{7}	11.32
	-4	1.133×10^{8}	1.136×10^{8}	11.36
	-3	1.148×10^{9}	1.150×10^{9}	11.50
	-2	1.165×10^{10}	1.165×10^{10}	11.65
	-1	1.173×10^{11}	1.173×10^{11}	11.73
	0	1.176×10^{12}	1.176×10^{12}	11.76
	1	1.177×10^{13}	1.177×10^{13}	11.77
5500	-5	1.291×10^{7}	1.326×10^{7}	13.26
	-4	1.299×10^{8}	1.317×10^{8}	13.17
	-3	1.307×10^{9}	1.316×10^{9}	13.16
	-2	1.322×10^{10}	1.326×10^{10}	13.26
	-1	1.340×10^{11}	1.341×10^{11}	13.41
	0	1.349×10^{12}	1.349×10^{12}	13.49
	1	1.352×10^{13}	1.352×10^{13}	13.52
6000	-5	1.405×10^{7}	1.546×10^{7}	15.46
	-4	1.457×10^{8}	1.530×10^{8}	15.30
	-3	1.477×10^{9}	1.513×10^{9}	15.13
	-2	1.490×10^{10}	1.507×10^{10}	15.07
	-1	1.508×10^{11}	1.515×10^{11}	15.15
	0	1.523×10^{12}	1.526×10^{12}	15.26
	1	1.531×10^{13}	1.531×10^{13}	15.31

---

* Based on species concentration from calculations by Hochstim [40,54]

TABLE D-1 (Cont.)

[illegible]	$\log_{10}\rho/\rho_0$	$\bar{\nu}$, [illegible], sec^{-1}	$\bar{\nu}$, sec^{-1}	[illegible]
4500	-5	1.320×10^{7}	1.720×10^{7}	[illegible]
	-4	1.541×10^{8}	1.752×10^{8}	[illegible]
	-3	1.626×10^{9}	1.733×10^{9}	[illegible]
	-2	1.658×10^{10}	1.710×10^{10}	[illegible]
	-1	1.678×10^{11}	1.701×10^{11}	[illegible]
	0	1.696×10^{12}	1.705×10^{12}	[illegible]
	1	1.708×10^{13}	1.710×10^{13}	[illegible]
5000	-5	1.008×10^{7}	1.938×10^{7}	[illegible]
	-4	1.442×10^{8}	1.921×10^{8}	19.21
	-3	1.701×10^{9}	1.948×10^{9}	19.48
	-2	1.803×10^{10}	1.924×10^{10}	19.24
	-1	1.843×10^{11}	1.898×10^{11}	18.98
	0	1.865×10^{12}	1.887×10^{12}	18.87
	1	1.879×10^{13}	1.885×10^{13}	18.85
5500	-5	0.851×10^{7}	3.423×10^{7}	34.23
	-4	1.173×10^{8}	2.159×10^{8}	21.59
	-3	1.631×10^{9}	2.113×10^{9}	21.13
	-2	1.886×10^{10}	2.124×10^{10}	21.24
	-1	1.985×10^{11}	2.095×10^{11}	20.95
	0	2.023×10^{12}	2.069×10^{12}	20.69
	1	2.041×10^{13}	2.055×10^{13}	20.55
6000	-5	0.8803×10^{7}	8.300×10^{7}	83.00
	-4	9.088×10^{8}	3.293×10^{8}	32.93
	-3	1.422×10^{9}	2.301×10^{9}	23.01
	-2	1.868×10^{10}	2.282×10^{10}	22.82
	-1	2.083×10^{11}	2.277×10^{11}	22.77
	0	2.163×10^{12}	2.245×10^{12}	22.45
	1	2.191×10^{13}	2.217×10^{13}	22.17
7000	-5	1.032×10^{7}	4.238×10^{8}	423.8
	-4	1.056×10^{8}	1.308×10^{9}	130.8
	-3	1.153×10^{9}	4.657×10^{9}	46.57
	-2	1.558×10^{10}	2.694×10^{10}	26.84
	-1	2.075×10^{11}	2.540×10^{11}	25.40
	0	2.345×10^{12}	2.542×10^{12}	25.42
	1	2.439×10^{13}	2.509×10^{13}	25.09
8000	-5	1.126×10^{7}	1.451×10^{9}	1451
	-4	1.203×10^{8}	4.356×10^{9}	435.6
	-3	1.242×10^{9}	1.333×10^{10}	133.3
	-2	1.370×10^{10}	4.780×10^{10}	47.80
	1	1.835×10^{11}	2.871×10^{11}	28.71
	0	2.350×10^{12}	2.732×10^{12}	27.32
	1	2.592×10^{13}	2.730×10^{13}	27.30

TABLE D-1 (Cont'd)

[illegible]	$\log_{10}\rho/\rho_0$	$\nu_{e,i}$, sec^{-1}	$\bar{\nu}$, sec^{-1}	[illegible]
9000	-5	1.030×10^{7}	3.465×10^{9}	3465
	-4	1.285×10^{8}	1.097×10^{10}	1097.
	-3	1.381×10^{9}	3.269×10^{10}	326.9
	-2	1.441×10^{10}	1.015×10^{11}	101.5
	-1	1.658×10^{11}	4.040×10^{11}	40.40
	0	2.207×10^{12}	2.911×10^{12}	29.11
	1	2.639×10^{13}	2.873×10^{13}	28.73
10000	-5	6.871×10^{6}	5.872×10^{9}	5872
	-4	1.218×10^{8}	2.164×10^{10}	2164.
	-3	1.472×10^{9}	6.658×10^{10}	665.8
	-2	1.570×10^{10}	1.981×10^{11}	198.1
	-1	1.672×10^{11}	6.577×10^{11}	65.77
	0	2.048×10^{12}	3.348×10^{12}	33.48
	1	2.594×10^{13}	2.966×10^{13}	28.66

APPENDIX E

AVERAGE ELECTRON COLLISION FREQUENCIES FOR WEAKLY IONIZED He, Ar, Kr and Xe

In all Tables (Appendix E and F) only two significant digits can be expected to be accurate, the remaining two digits are included for smoothness and for comparison studies of effects of change in cross sections on collision frequency.

TABLE E-1

AVERAGE ELECTRON COLLISION FREQUENCY FOR WEAKLY IONIZED HELIUM

$(\omega >> \nu)$

$$\nu = \bar{\nu}_0 \cdot 10^{11} \frac{\rho}{\rho_0} = \bar{\nu}_1 \cdot 10^8 P_{mmHg} \quad , \quad sec^{-1}$$

[illegible]	$\bar{\nu}_0$	[illegible]
100	1.168	[illegible]
200	1.715	[illegible]
300	2.164	[illegible]
400	2.563	2.303
500	2.929	2.105
600	3.268	1.958
700	3.584	1.840
800	3.880	1.743
900	4.159	1.661
1000	4.421	1.589
1500	5.554	1.331
2000	6.479	1.164
2500	7.268	1.045
3000	7.961	0.9538
3500	8.581	0.8812
4000	9.141	0.8214
4500	9.653	0.7710
5000	10.12	0.7276
6000	10.95	0.6562
7000	11.67	0.5991
8000	12.29	0.5522
9000	12.84	0.5126
10000	13.32	0.4788
11000	13.76	0.4496
12000	14.15	0.4240
13000	14.52	0.4014
14000	14.85	0.3812
15000	15.16	0.3632
16000	15.44	0.3469
17000	15.71	0.3322
18000	15.96	0.3187
19000	16.20	0.3064
20000	16.42	0.2950

TABLE E-[illegible]

AVERAGE ELECTRON COLLISION FREQUENCY FOR WEAKLY IONIZED ARGON

$(\alpha \gg \nu)$

$$\bar{\nu} = \bar{\nu}_0 \cdot 10^{11} \frac{\rho}{\rho_0} = \bar{\nu}_1 \cdot 10^{8}\, p_{mmHg} \quad , \quad sec^{-1}$$

T, °K	$\bar{\nu}_0$	$\bar{\nu}_1$
100	0.8358	[illegible]
200	0.6865	[illegible]
300	0.5469	[illegible]
400	0.4410	[illegible]
500	0.3657	0.262
600	0.3130	0.1875
700	0.2767	0.1410
800	0.2527	0.1135
900	0.2381	0.0951
1000	0.2311	0.0831
1500	0.2791	0.0669
2000	0.4234	0.0761
2500	0.6350	0.0913
3000	0.8974	0.1075
3500	1.201	0.1234
4000	1.541	0.1385
4500	1.914	0.1529
5000	2.317	0.1665
6000	3.207	0.1921
7000	4.199	0.2156
8000	5.284	0.2374
9000	6.452	0.2577
10000	7.693	0.2765
11000	9.000	0.2941
12000	10.36	0.3104
13000	11.78	0.3256
14000	13.23	0.3396
15000	14.72	0.3526
16000	16.23	0.3645
17000	17.76	0.3754
18000	19.29	0.3852
19000	20.83	0.3941
20000	22.37	0.4020

TABLE E-3

AVERAGE ELECTRON COLLISION FREQUENCY FOR WEAKLY IONIZED KRYPTON

$(\omega >> \nu)$

$$\bar{\nu} = \bar{\nu}_0 \cdot 10^{11} \frac{p}{p_0} = \bar{\nu}_1 \cdot 10^8 P_{mmHg} \quad , \quad sec^{-1}$$

$T, {}^\circ K$	$\bar{\nu}_0$	$\bar{\nu}_1$
100	4.187	15.05
200	4.106	7.380
300	3.854	4.617
400	3.568	3.206
500	3.277	2.356
600	2.999	1.797
700	2.744	1.409
800	2.514	1.129
900	2.310	0.9225
1000	2.131	0.7660
1500	1.529	0.3664
2000	1.244	0.2235
2500	1.141	0.1640
3000	1.162	0.1392
3500	1.282	0.1317
4000	1.469	0.1338
4500	1.774	0.1417
5000	2.135	0.1534
6000	3.060	0.1833
7000	4.232	0.2173
8000	5.618	0.2534
9000	7.188	0.2871
10000	8.916	0.3205
11000	10.78	0.3522
12000	12.75	0.3820
13000	14.82	0.4098
14000	16.97	0.4357
15000	19.18	0.4596
16000	21.44	0.4817
17000	23.74	0.5020
18000	26.07	0.5206
19000	28.42	0.5376
20000	30.77	0.5530

TABLE E-4

AVERAGE ELECTRON COLLISION FREQUENCY
FOR WEAKLY IONIZED XENON

$(\omega >> \nu)$

$$\bar{\nu} = \bar{\nu}_0 \cdot 10^{11} \frac{\rho}{\rho_0} = \bar{\nu}_1 \quad 10^{8} P_{\mathrm{mmHg}} \quad , \quad \mathrm{sec}^{-1}$$

T, °K	$\bar{\nu}_0$	$\bar{\nu}_1$
100	17.20	61.81
200	15.24	27.39
300	13 45	16.11
400	11.89	10.79
500	10.77	7 745
600	9.734	5.831
700	8.833	4.536
800	8.051	3.617
900	7 369	2.943
1000	6.776	2 436
1500	4 804	1.151
200	3.924	0.7052
2500	3.709	0.5332
3000	3.953	0.4736
3500	4.550	0.4672
4000	5.440	0.4888
4500	6.596	0.5260
5000	7.958	0.5721
6000	11.29	0.6761
7000	15.24	0 7826
8000	19.66	0.8333
9000	24.39	0.8742
10000	29 32	1.054
11000	34.33	1.122
12000	39.36	1.179
13000	44.33	1 226
14000	49.21	1.263
15000	53 97	1.293
16000	58.57	1 316
17000	63.02	1 332
18000	67.28	1 344
19000	71 38	1.350
20000	75.28	1.353

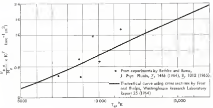

FIGURE E-1 Average Electron Collision Frequency for Xenon ($\omega >> \bar{\nu}$)

APPENDIX F

DC ELECTRIC CONDUCTIVITY FOR O_2, N_2 AND DRY AIR ($T < 2000^\circ K$)

TABLE F-1
THE DC ELECTRIC CONDUCTIVITY OF O_2 AND AIR
(MODEL I)

$$\sigma' = \frac{\omega_p^2}{\nu_{ec}}$$

$$\nu_{ec} = \nu^0_{ec} \cdot 10^{11} \frac{\rho}{\rho_0}\ ,\ \nu_{ec} = \frac{\bar{\nu}}{\alpha_e(0)\alpha_s(0)}\ ,\ \alpha_e(0) = \alpha_e \text{ at } x = 0, \text{ etc.}$$

T, °K	ν^0_{ec} for O_2, (Model I)	ν^0_{ec} for Air, (Model I)
200	0.625	0.778
250	0.734	0.954
300	0.840	1.130
350	0.945	1.305
400	1.050	1.479
500	1.255	1.824
600	1.457	2.163
700	1.656	2.494
800	1.851	2.818
900	2.043	3.134
1000	2.232	3.444
1200	2.599	4.041
1400	2.952	4.610
1600	3.291	5.156
1800	3.616	5.681
2000	3.927	6.187

TABLE F-2

THE DC ELECTRIC CONDUCTIVITY FOR DIATOMIC NITROGEN

$$\sigma_{ec} = \frac{\omega_p^2}{\nu_{ec}}$$

$$\nu_{ec} = \nu^0_{ec} \cdot 10^{11} \frac{\rho}{\rho_0} \text{ , sec}^{-1}$$

T, °K	ν^0_{ec}
250	1.024
300	1.209
350	1.405
400	1.600
500	1.985
600	2.365
700	2.736
800	3.099
1000	3.799
1200	4.465
1400	5.101
1600	5.710
1800	6.295
2000	6.860

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APPENDIX G REVIEW OF ELASTIC CROSS SECTIONS

The cross sections below 1 ev are known to have a rather violent history, both in the value and in the shape, which only after prolonged and repeated measurements by different methods and detailed calculations converge to some accepted value.

All computations in the appendices were performed using numerical integration method (Simpson's) and the cross sections were given at a closely spaced even interval (see Table G-1) and interpolated wherever needed in numerical integration. Formulae for cross section for N_2 were used[3] for a special energy range.*

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* References are at the end of this Appendix

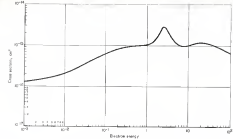


FIGURE G-1 Electron Momentum Transfer Cross Section for N_2
(from Phelps, Ref 4)

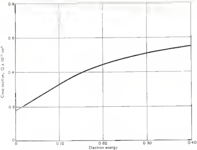
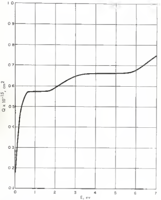


FIGURE G 2 Electron Momentum Transfer Cross Section for O_2 (from Ref 1)



(Based on Refs. 1, 2 and Bruche Ref. 8)

FIGURE G-3 Electron Momentum Transfer Cross Section for O_2

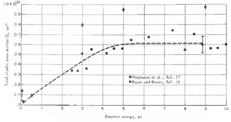


FIGURE G-4. Total Elastic Cross Section for Electrons with Atomic Nitrogen

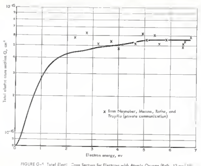


FIGURE G-5 Total Elastic Cross Section for Electrons with Atomic Oxygen (Refs 12 and 1

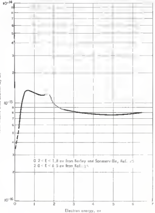


FIGURE G-6 Total Elastic Cross Section for Electrons with NO

Comments on the Adapted Values (for the numerical integration) of Electron Momentum Transfer Cross Sections.

For O_2

$0.025 \leq E \leq 1.8$ ev	From review by Shkarofsky, Bachynski, and Johnson[1] (1961). [Based on private communication to Shkarofsky et al. by R.W. Crompton and L.G.H. Huxley (subject to a correct distribution) using data of R.A. Nielsen and N.E. Bradbury, Phys. Rev. 51, 69 (1937).] The numbers were read from Fig. 2a in Ref. 1 and extrapolated to energies below 0.025 ev
$1.8 \leq E \leq 6.8$ ev	Bruche[8] (1927). Shape verified by Meynaber et al [14] (1961).
For He, Xe, Kr	Based on cross sections by Frost and Phelps[17]

For N_2

$3 \times 10^{-3} < E < 0.05$ ev	Data by Pack and Phelps[3] (1961) are given by the formula:

For N_2 (cont):

$$Q = \frac{10^{-16}\ \sqrt{E_{eV}}\ [illegible]}{0.531 - 3.13\times10^{-[illegible]}/E_{eV}}$$

[illegible] gives $Q = 0.426\times10^{-16}\,cm^2$ at E [illegible]

$Q = 0.129\times10^{-16}$ at E = 0.003 [illegible]

$0 < E < 3\times10^{-3}$ ev

By taking straight line approximation, so that its slope is equal to the slope given by above formula at E = 0.003 ev, we obtained

$Q = 0.0945\times10^{-16}$ at E = 0

$0.1 < E < 1.75$ ev

Based on R.W. Crompton and L.G.H. Huxley [private communication to Shkarofsky et al (1961)], who used data of R.A. Nielsen, Phys. Rev. 50, 950 (1936). Read from graph in Ref. 1.

$0.01 < E \leq 7$ ev

Recent data by Phelps[4,16] (1963 to 1964) are in perfect agreement with above (see slight departure in our Fig. 1, which has no noticeable effect on the collision frequency).

For O

$0 \leq E \leq 2$ ev

From graph in Bates and Massey[12] (1947), with added p-wave contribution from Klein and Brueckner[13] (1958), according to Neynaber et al[14] (1961), through a point by Lin and Kivel (1959).

[illegible] E [illegible] ev

The above theoretical data was joined smoothly to the following cross-section:

[illegible] ≤ [illegible] ev

From experiment by Neynaber, Marino, Rothe and Trujillo[14] (1961), (measured by $2.3 \leq E \leq 11.5$ ev)

$Q = (5.5 \pm 0.4) \times 10^{-16}\ cm^2$.

	(Actually they measured the ratio of $Q(O)/Q(O_2)$ and used values for $Q(O_2)$ of E. Bruche[5] (1927).
For N	
2.5 - 10 ev	Neynaber, Marino, Rothe and Trujilo[7] (1963) [actually measured ratio of $Q(N)/Q(N_2)$ and used $Q(N_2)$ from Normand[11]]
0.1 - 0.5 ev	Bauer and Browne[8] (1963)
0.5 - 2 ev	Joined smoothly between Refs 7 and 8
For NO:	
0.2 ev - 1.8 ev	Based on Bailey and Sommerville[15] (1934), chosen on recommendation of Ref. 1 (their curve 6).
0 - 0.2 ev	Extrapolated from above.
2 ev - 6.5 ev	Read from graph in Massey and Burhop[5] (1956). Based on data obtained by Ramsauer method.
1.8 ev - 2.0 ev	Joined smoothly between values from Refs. 5 and 15
For A	
0 < E	Frost and Phelps[3] (1962), and Engelhardt and Phelps[6] (1964). Argon was used for air for T < 2000° only.

TABLE C-1

ADAPTED FOR CALCULATIONS
ELECTRON-NEUTRAL COLLISION CROSS SECTIONS FOR MOMENTUM
TRANSFER AS A FUNCTION OF ELECTRON ENERGY

(Q in units of $10^{-15} cm^2$)

[illegible]	$Q(O_2)$	$Q(N_2)$	Q(NO)	Q(N)	Q(O)
0.00	(0.177)	(0.0945)	(0.30)	0.200	0.0792
0.05	0.255	0.425	(0.35)	.050	.0800
0.10	0.333	0.625	(0.40)	.020	.0820
0.15	0.398	0.720	(0.46)	.030	.0880
0.20	0.437	0.789	0.525	0.038	0.0924
0.25	0.475	0.839	0.626	.048	.1000
0.30	0.505	0.875	0.805	.056	.1080
0.35	0.525	0.907	0.890	.062	.1170
0.40	0.545	0.937	1.015	0.073	0.1275
0.45	0.558	0.957	1.124	.085	.1390
0.50	0.570	0.975	1.220	.095	.1500
0.55	0.575	0.990	1.288	.100	.1610
0.60	0.579	1.005	1.320	0.105	0.1716
0.65	0.577	1.020	1.335	.113	.1840
0.70		1.032	1.340	.121	.1950
0.75		1.042	1.337	.129	.2050
0.80		1.052	1.327	0.135	0.2156
0.85		1.065	1.315	.142	.2280
0.90		1.080	1.258	.150	.2410
0.95		1.095	1.272	.158	.2530
1.00		1.115	1.268	0.164	0.2640
1.05		1.130	1.254	.171	.2780
1.10		1.152	1.240	.179	.2930
1.15		1.172	1.228	.184	.3080
1.20		1.195	1.218	0.190	0.3212
1.25		1.217	1.210	.198	.3320
1.30		1.240	1.206	.203	.3420
1.35		1.262	1.204	.209	.3510
1.40		1.282	1.204	0.217	0.3608
1.45		1.305	1.206	.220	.3690
1.50		1.325	1.215	.227	.3770
1.55		1.344	1.222	.231	.3840
1.60	↓	1.410	1.230	0.239	0.3916
1.65		1.490	1.238	.246	.3980
1.70	0.577	1.570	1.250	.251	.4020

E(ev)	[illegible]	Q(N$_2$)	Q(NO)	[illegible]	Q(O)
1 75	[illegible]	1.640	1 264	260	407
1.80	0 577	1.700	1.276	0.267	0 4092
1.85	0.585	1 78	1.25	271	416
1.90	0.587	1.82	1.075	279	419
1 95	0.590	1.92	1.025	.287	421
2 00	0.592	2 00	0.98	0 294	0.422
2 05	0.596	2.10	0.95	300	428
2 10	0.599	2.21	0 93	307	430
2 15	0.603	2 32	0.915	.312	434
2 20	0.606	2.40	0 90	0.319	0.440
2 25	0.610	2.45	0.881	327	440
2 30	0.613	2 48	0 88	331	441
2 35	0.616	2.49	0 875	340	441
2 40	0 619	2.50	0.87	0.348	0 442
2.45	0.622	2 50	0 865	351	443
2 50	0.625	2.50	0.860	359	445
2 55	0.628	2.49	0.855	.368	447
2.60	0.631	2.46	0.850	0.373	0 449
2 65	0.633	2 43	0 850	379	451
2 70	0.636	2.39	0 849	.385	454
2 75	0 638	2.35	0 840	.391	.456
2.80	0 640	2 29	0 835	0.400	0.458
2.85	0 642	2.23	0 832	.406	.463
2 90	0 644	2.16	0.830	411	.468
2.95	0 646	2.08	0 825	419	.470
3 00	0 647	2 00	0 820	0.426	0.475
3 05	0.648	1.93	0 817	431	.476
3.10	0 650	1.86	0.815	440	478
3 15	0 651	1 80	0 813	447	479
3.20	0 653	1.75	0 810	0 452	0.480
3 25	0 654	1.69	0.809	460	481
3.30	0 655	1 64	0 805	467	482
3 35	0 656	1 58	0.802	472	485
3 40	0 657	1 53	0.800	0.480	0.488
3 45	0.658	1.48	0.795	.485	489
3.50	0 658	1 43	0.790	492	490
3 55	0.659	1 37	0 790	497	491
3.60	0.659	1 32	0.790	0.503	0.493
3 65	0.659	1.26	0.790	510	.495
3.70	0 660	1.22	0 790	516	.497
3 75	0 660	1 18	0.790	520	499
3 80	0 660	1 13	0.785	0 527	0.501
3 85	0.660	1.09	0.780	530	503
3.90	0 660	1 06	0.775	535	506
3 95	0.660	1.00	0 770	540	.508
4.00	0.660	990	0.766	0.545	0 510
4 05	0.660	980	0 766	0 550	0.510

[illegible]	Q(O$_2$)	Q(N$_2$)	Q(NO)	Q(N)	Q(O)
4.10	0.660	.960	0.766	0.5555	0.511
4.15	0.660	.960	0.766	.559	.511
4.20	0.660	.950	0.766	0.563	0.512
4.25	0.660	.950	0.766	.567	.516
4.30	0.660	.950	0.766	.567	.519
4.35	0.660	.950	0.766	.570	.523
4.40	0.660	.950	0.766	0.577	0.526
4.45	0.660	.950	0.766	.580	.528
4.50	0.660	.950	0.766	.584	.530
4.55	0.660	.950	0.766	.587	.533
4.60	0.660	.960	0.766	0.588	0.536
4.65	0.660	.960	0.766	.591	.538
4.70	0.660	.960	0.766	.593	.540
4.75	0.660	.970	0.7625	.597	.542
4.80	0.660	.970	0.7650	0.598	0.544
4.85	0.660	.980	0.7675	.599	.547
4.90	0.660	.980	0.770	.600	.548
4.95	0.660	.990	0.7725	.601	.549
5.00	0.660	1.00	0.775	0.601	0.550
5.05	0.660	.990	0.7775	.601	.550
5.10	0.660	.990	0.780	.602	.550
5.15	0.661	.990	0.782	.602	.550
5.20	0.661	.990	0.7835	0.602	0.550
5.25	0.6615	.990	0.785	.602	.550
5.30	0.662	.990	0.7865	.602	.550
5.35	0.6625	.990	0.788	.602	.550
5.40	0.663	.980	0.790	0.602	0.550
5.45	0.664	.980	0.790	.602	.550
5.50	0.6646	.980	0.791	.602	.550
5.55	0.6653	.980	0.7915	.603	.550
5.60	0.666	.980	0.792	0.603	0.550
5.65	0.668	.980	0.7925	.603	.550
5.70	0.6695	.980	0.793	.603	.550
5.75	0.671	.980	0.794	.603	.550
5.80	0.673	.980	0.795	0.604	0.550
5.85	0.675	.980	0.7975	.604	.550
5.90	0.677	.980	0.800	.604	.550
5.95	0.680	.980	0.8025	.604	.550
6.00	0.683	.980	0.805	0.604	0.550
6.05	0.686	.980	0.8075	.604	.550
6.10	0.689	.980	0.810	.605	.550
6.15	0.693	.980	0.810	.605	.550
6.20	0.696	.980	0.810	0.605	0.550
6.25	0.701	.980	0.810	.605	.550
6.30	0.705	.980	0.810	.605	.550
6.35	0.709	.990	0.810	.606	.550
6.40	0.713	.980	0.810	0.606	0.550

E(ev)	$Q(O_2)$	$Q(N_2)$	Q(NO)	Q(N)	Q(O)
6.45	0.717	.980	0.81	0.607	0.550
6.50	0.722	.980	0.81	.608	.550
6.55	0.726	.980	0.81	.608	.550
6.60	0.730	.980	0.81	0.608	0.550
6.65	0.734	.980	0.81	.609	.550
6.70	0.738	.990	0.81	.609	.550
6.75	0.742	.990	0.81	.610	.550

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CURRICULUM VITAE

Adolf R. Hochstim was born in Krakow (Cracow), Poland on November 13, 1928. From 1946 to 1949 he studied physics, mathematics and chemistry at J. W. Goethe University in Frankfurt a.M., West Germany, under Professor E. Madelung, et al. He emigrated with his parents to the United States in August 1949. From 1950 to 1951 he attended graduate courses at University of Miami and received his B.S. in Physics. As a graduate teaching assistant in Physics at Florida State University (1951-1953) he received his M.S. in Physics[1] in 1953 under Professor R. Frank. From 1953 to 1956 as a graduate teaching assistant in Physics he attended the University of Florida, completed all course work, passed all qualifying examinations and oral examination on the thesis[2], which he did not submit for final approval. At that time he accepted employment at the Physics Section at Convair (a division of General Dynamics Corporation) in San Diego, California where he became interested in the applications of physics to reentry of space vehicles[3-9] in research of high temperature gases (statistical mechanics and thermodynamics)[9-12, 14,15,17-23], gravitational-Doppler shift[13] and translation of plasma physics book from Russian[16].

In 1963 he, jointly with J. Gilvarry[24], postulated a new theory of origin of organic compounds (hypersonic chemisynthesis) which he further extended[25]. His interest in plasma physics and microwave propagation through plasma led to research[28,30,31,32,33,34] and book writing[36,37,38,39]. In 1964, after the elapsed time of the status of limitations, he returned for a few months to the

University of Florida, where he gave a series of seminars on Microwave Propagation through Plasma, and in December 1966 submitted a new thesis, which will be published as Chapter I in a book by Academic Press[36].

Since 1962 he has been a staff member of the Institute for Defense Analyses in Arlington, Virginia where he is engaged in various research activities.

He is a member of Sigma Xi, American Physical Society, Combustion Institute, American Institute of Aeronautics and Astronautics, American Astronautics Society and the Philosophical Society of Washington, D.C.

LIST OF PUBLICATIONS IN UNCLASSIFIED LITERATURE
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This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Arts and Sciences and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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